

CSE 20—Discrete Math

Winter, 2006

January 10 (Day 1)
Introduction
Boolean Functions

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Miscellaneous

See website:

[<www.cse.ucsd.edu/classes/wi06/cse20>](http://www.cse.ucsd.edu/classes/wi06/cse20)

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Administrative Details

Homework

- Assigned every week
- Need not be turned in

Reading

- Two textbooks
 - Tucker
 - Solow
- Read readings *before* class
- Print slides *before* class (available by 6 p.m. the previous day)
- Take notes on slides *during* class
- Read slides (including notes) *after* class
 - Read slides online with my inked markup
- Read readings again, *after* class

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Administrative Details

Quizzes

- On Thursdays
- Taken *almost* directly from homework
 - Slight changes to numbers, etc.

Midterms

- Two: Feb. 2 and Feb. 28

Final

Practice, practice, practice!

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Overview

This course provides the mathematical background necessary for reasoning in computer science. Students will learn logic and basic proof techniques useful for the design and verification of programs and circuits.

CSE 20:

- Boolean functions and Computer Arithmetic
- Logic
- Number Theory/Cryptography
- Sets/Functions
- Induction/Sequences/Series
- Equivalence/Order

CSE 21

- Combinatorics
- Recurrence Relations
- Generating Functions
- Decision Trees
- Graph Theory

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Discrete Mathematics

Continuous mathematics:

- What are the dimensions of a cylindrical soup can that minimizes the amount of metal used?
- Fractional knapsack problem
 - Various items of loot
 - Each item has a particular weight and value
 - Thief has a knapsack that'll hold an overall weight
 - Thief can take as much of an item as desired

Discrete mathematics

- Usually dealing with integers
 - Often makes the problem harder!
- Knapsack problem
 - Thief must take all of an item or none

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Discrete Math is Mostly about Counting

Examples

- How many ways to choose a pair of people in this class
- How many pins in a generalized bowling game

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Examples

How many binary trees with a certain number of nodes?

How many ways to multiply together n matrices (using pairwise multiplication)?

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Rule of Product

If we're looking at outcomes by making a sequence of k choices such that:

- The i th choice can be made in c_i ways
 - c_i independent of previous choices
- Each outcome can occur in only one way
 - outcomes are all distinct

then the number of outcomes is $c_1 \cdot c_2 \cdot \dots \cdot c_n$

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Example of Rule of Product

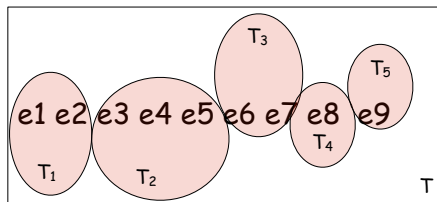
There are five Prom King candidates and six Prom Queen candidates. How many ways are there to pick a pair of Prom King and Queen?

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Rule of Sum

If a set of T elements can be partitioned into sets $T_1, \dots, T_j$, such that each element in T appears in exactly one set T_i , then:

- $|T| = |T_1| + |T_2| + \dots + |T_j|$



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Solved Example

Examples

- How many ways to choose a pair of people in this class

- How many pins in a generalized bowling game

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Computing Triangular Numbers

Graphically

Gauss's approach

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Boolean Functions

Boolean Functions

Give two sets A and B, a function is a rule that tells how to find a unique $b \in B$ for each $a \in A$.

- It is shown as $f:A \rightarrow B$
- A is the *domain*
- B is the *range* (or *codomain*)

Boolean functions map from $n \{0, 1\}$ values to $\{0, 1\}$

- How many boolean functions are there that take 0 inputs?
- 1 input?
- 2 inputs?

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Truth Tables

Contain one row for each combination of possible inputs, along with output

- p
- NOT p
 - $\sim p$
 - $\neg p$
- p AND q
 - $p \wedge q$
 - pq
- p OR q
 - $p \vee q$
 - $p + q$
- p XOR q
 - $p \oplus q$

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Constructing Boolean Functions

Operators:

- Unary:
 - $\sim$
- Binary (common):
 - $\wedge$ (AND), $\vee$ (OR), $\oplus$ (XOR)
- Precedence: Unary not has higher precedence than binary operators
 - AND has precedence over OR

Equivalence: Two boolean functions are equal if and only if they have the same truth table

- Example: $\sim(pq + \sim p \sim q) =$

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Truth Table $\rightarrow$ Boolean Function

Disjunctive Normal Form

a	b	c	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

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How Computers Add

We want to add two binary numbers:

```
13
+22
----
35
```

Rules:

- $0 + 0 =$
- $0 + 1 =$
- $1 + 0 =$
- $1 + 1 =$

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Half Adder (Single-bit Adder)

Truth Table:

p	q	s	c
0	0		
0	1		
1	0		
1	1		

Black box:

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Half Adder

Calculating the carry bit

p	q	S	C
0	0		
0	1		
1	0		
1	1		

Circuit:

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Half Adder

Calculating the sum bit

A	B	S	C
0	0		
0	1		
1	0		
1	1		

Circuit:

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Half Adder

A	B	S	C
0	0		
0	1		
1	0		
1	1		

Overall circuit:

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Other Gates

Or

Invert (not)

Nand

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Combinatorial (Combinational) Circuit

An input wire can split to be used as input to two gates

An output wire can be used as input

The outputs depend only on the present inputs

- Don't depend on history
- One way to achieve this: no cycles
- Sequential circuits: output depends on the history

Direct correspondence between combinatorial circuits and boolean functions

- Master's thesis by Claude Shannon: 1937

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Full Adder

Half-adder only works for rightmost bits of two numbers.

Full adder can also handle the incoming carry bit

Circuit?

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