CSE 203 Homework 1
Approximation algorithms, basic techniques
Due Wed., Feb. 4

General instructions: Each problem is worth 10 points. For each of the algorithm problems, design a polynomial-time approximation algorithm with a performance ratio as indicated. Give a proof of the approximation guarantee and time analysis for each algorithm. You may use any well-known algorithm or data structure, or algorithm from the texts (including CLRS) or from class, as a sub-routine, without needing to provide details. You can use the NP completeness of any problem proved NP complete in CLRS or in *Garey and Johnson*, for problem 3.

Vazirani, Ex. 3.2, pages 33-34. Only give the algorithm that guarantees ratio 2; you don't need to prove NP-hardness or give a poly-time algorithm when $S \cup R = V$.

The *Minimum Average Finish Time Schedule on Two Machines* is as follows: You are given a list of n jobs of positive real durations $d_1, ..d_n$. You must assign each job to one of two identical machines, and order the jobs on each machine. The *finish time* of a job i is the sum of all the durations of all jobs assigned before it on the same machine, plus its own duration. The objective is to minimize the sum of finish times of all jobs. Give a FPTAS for this problem. (Hint: first, give an optimal algorithm for the single machine case. Then use this to give a dynamic programming pseudo-polynomial time algorithm for the case when all durations are small integers. Reduce the general case to the small integer case.)

The *bin covering* problem is as follows: The instance is a list of n items, each with a positive real *size*, $a_i > 0$. There are an unlimited number of identical bins, each of size 1. A solution places each item into a bin. A bin is *full* if the sum of the sizes of the items in the bin is ≥ 1. The objective is to *maximize* the number of full bins. (This problem is also defined in, Vazirani, p. 77, problem 9.9). A. Give a factor 2 approximation algorithm for this problem. (6 points). B. Prove that if an algorithm with ratio $\gamma < 2$ exists, then $P = NP$. (4 pts. Hint: reduce from the known NP-complete problem *partition*: Given a list of positive integers, $a_1, ...a_n$, is there a subset $S \subseteq \{1, ..n\}$ with $\sum_{i \in S} a_i = \sum_{i \notin S} a_i$?)

The *Interval scheduling with proportional profit* problem is the following: You are given a set of jobs, j_i, each with a start time s_i and finish time $f_i > s_i$, and a number m of identical machines. The *duration* of a job is $d_i = f_i - s_i$. For each job, a schedule specifies either a machine $1 \leq i \leq m$ to schedule it on, or that it isn't scheduled. A legal schedule never schedules two jobs whose intervals (s_i, f_i) intersect on the same machine, i.e., machines can only perform one job at a time. The objective is to maximize the total duration of the scheduled jobs. Give a 3-approximation algorithm for this problem. (This problem is also known as Maximum Throughput interval scheduling, since we are maximizing the total work done.)