CSE 203 Final Exam
Randomized Algorithms, Approximation, and Linear Programming Relaxation
Due March 17, 11:59 PM.

Answer **3 of the 4** problems below. Each problem is worth 10 points. For each of the algorithm problems, design a polynomial-time algorithm with the best possible approximation ratio, or other standard as given in the problem. Your algorithm can be randomized or deterministic. Several problems have multiple solutions, with different ratios or other quality guarantees.

For each algorithm, give a correctness argument (explanation, if it is relatively simple, or detailed proof if not) and time analysis. You may use any well-known algorithm or data structure, or algorithm from the text or from class, as a sub-routine without needing to provide details.

Smallest Hamming ball containing strings: Let $x \in \{0,1\}^m$ and let $0 \leq d \leq m$ be an integer. The Hamming ball of radius d centered at x ($H_d(x)$) is the set of all strings of length m differing from x in at most d positions. The problem is, given n strings $y_1, ..y_n \in \{0,1\}^m$, find a Hamming ball $H_d(x)$ containing all the strings, so that d is as small as possible. First, give a (almost trivial)factor two approximation algorithm for this problem. (3 points) Then show how, if d_{min} is the optimal solution, we can find (with high probability) a Hamming ball containing the given strings of radius $d \leq d_{min} + O(\sqrt{d_{min} \log n})$. (7 pts. Hint: use linear programming relaxation with randomized rounding. You can use Chernoff bounds: Theorem 4.1 on page 68 of MR, together with Theorem 4.3 and inequality 4.12 (it takes a while to follow all the pointers connecting these, but that should allow you to avoid most of the messy calculations.) Note that if $d_{min} \leq O(\log n)$, then part a. also satisfies part b.)

Satisfiable MAX-3SAT Consider the special case of $MAX-3SAT$ when the 3-CNF formula is satisfiable, i.e., when you know that the optimal solution satisfies all m clauses. Give the best approximation guarantee that you can for this case. It should be better than 3/4, since that is known for the general case. (Hint: Go back and forth between solving satisfiability and MAX-SAT. Either there are many clauses with at most 2 variables, or few such clauses. You might want to use different approaches to handle these two cases.)

Degree 3 vertex cover Consider the vertex cover problem restricted to unweighted degree 3 graphs, i.e., all nodes have at most 3 neighbors and the objective is to minimize the size of the vertex cover. (Hint: The complement of an independent set is a vertex cover. There is a relatively easy 5/3 approximation algorithm, but more sophisticated methods might yield a better ratio. Since we have a ratio 2 algorithm for the general problem, you should definitely get a ratio less than 2 to get any credit.)

On-line commodity purchasing Consider the following "On-line commodity purchasing" problem. The state of California has a budget of B for energy purchases, and there are n companies it can buy energy from. For $1 \leq i \leq n$, Company i makes an offer to sell energy at a rate r_i, and the state needs to declare the amount a_i it will purchase from that company. It can't change this amount later (without a lawsuit and it usually loses.) The constraint is that the total purchases must fit within the budget, $\sum a_i r_i \leq B$, and the objective is to maximize the total amount purchased. The off-line solution is simply B/r_{min}, where r_{min} is the minimum rate r_i. Prove that there is a deterministic online algorithm that achieves approximation ratio $1/n$, and that any randomized online algorithm has competitive ratio $O(1/n)$. (Solutions that involve issuing bonds will not recieve any credit.)