CSE190 – Image Processing – Homework #8

Instructor: Prof. Serge Belongie. T.A.: Josh Wills.

`http://www-cse.ucsd.edu/~sjb/classes/wi02/cse190`

Due (in class) 1:25pm Wed. Mar. 13, 2002.

Written exercises

1. Suppose you are given a set of N feature vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ and k prototype vectors $\{\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_k\}$. Assume all of the vectors are of size $n \times 1$. Show that the $N \times k$ matrix of squared Euclidean distances between the feature vectors and the prototype vectors can be obtained by computing the following matrix:

$$T = \mathbf{E}\mathbf{1}_k^T + \mathbf{1}_N\mathbf{F}^T - 2X^TY$$

   where $\mathbf{E}$ is an $N \times 1$ vector with entries $E_i = \|\mathbf{x}_i\|^2$, $\mathbf{F}$ is a $k \times 1$ vector with entries $F_i = \|\mathbf{m}_i\|^2$, $\mathbf{1}$ is a column vector of ones whose length is given by its subscript, X is an $n \times N$ matrix of feature vectors, and Y is an $n \times k$ matrix of prototype vectors.

2. Consider the 2×2 matrix

$$A = \left[\begin{array}{cc} a & b \\ c & d \end{array} \right].$$

   Show that the inverse is given by

$$A^{-1} = \frac{1}{\det(A)} \left[\begin{array}{cc} d & -b \\ -c & a \end{array} \right].$$

Matlab exercises

1. Clustering in color space using k-means.

   (a) Implement the k-means clustering algorithm as described in class. For the stopping criterion, allow the user to supply a threshold on the change in J (the sum of squared error over all clusters) on each iteration, as well as a maximum allowed number of iterations (e.g. 25). Have your program output the value of J on each iteration. Initialize the cluster centers by choosing k feature vectors at random from the data.

   (b) Load in Figure 6.30(a) (the bowl of strawberries) and use `imresize` with the `'bilinear'` option and `M=0.25` to reduce its resolution by a factor of 4 in x and y. Display the resized image. Construct a matrix of 3-dimensional feature vectors for this image using the RGB values of each pixel.

   (c) Use k-means to perform clustering on this image using $k = 3$ and $k = 4$. In each case, run three trials to see the effects of the random initialization. Display each of the six resulting segmentations as a pseudocolor cluster membership image, like the example shown in class. (Note: if you don't have access to a color printer, it's ok to use shades of gray.)

   Things to turn in:

   - Code listing for parts 1a and 1b.
   - Program output and parameter settings for part 1c.

2. Lucas-Kanade optical flow.

(a) Implement the Lucas-Kanade algorithm for measuring optical flow, as described in class. Allow the user to specify the size of the window used in enforcing the smoothness constraint. Use the `quiver` function to display the optical flow vectors. In addition, have your program return the two eigenvalues of the windowed image second moment matrix at each pixel.

(b) Construct two frames of a simple motion sequence as follows. Make a 16×16 white square centered on a black background of size 32×32. Blur it with a Gaussian filter with $\sigma = 1$. This image represents $I(x, y, t)$. Produce the second image, representing $I(x, y, t + 1)$, by displacing the first image down one pixel and to the right one pixel. Display each frame, as well as I_t and the two components of ∇I.

(c) Compute and display the optical flow for the above sequence using a window size of 5×5. Since you know the "ground truth" displacement (i.e. $u = 1, v = 1$), comment on the accuracy of your measured optical flow at various points throughout the image. Demonstrate how, by applying a threshold on the eigenvalues, you can suppress the flow vectors at pixels that suffer from the aperture problem.

(d) Construct a new sequence consisting of the original first frame and a second frame produced by rotating the first one by 5° (use `imrotate` with the `'bil'` and `'crop'` options). Now repeat step 2c using this sequence.

Things to turn in:

- Code listing for steps 2a, 2b, 2c, and 2d.
- Program output for steps 2b, 2c, and 2d.
- Written comments for steps 2c and 2d.