CSE190 – Image Processing – Homework #1
Instructor: Prof. Serge Belongie. T.A.: Josh Wills.
`http://www-cse.ucsd.edu/~sjb/classes/cse190`
Due (in class) 1:25pm Wed. Jan. 16, 2002.

Reading

- Skim GW Ch. 1 and 2.
- GW 2.6, 3.5, 3.6.0–3.6.1, 3.7.0–3.7.1
- GW Review Material Ch. 3, "A Brief Overview of Linear Systems," up to the first occurrence of the word "Fourier." (Available at http://www.imageprocessingbook.com.)

Written exercises

1. GW, Problem 3.17.
2. GW, Problem 3.18.
3. GW, Problem 3.22.
4. Determine whether each of the following systems is linear and spatially invariant. Justify your answers.
   (a) $g(x) = e^{f(x)}$
   (b) $g(x) = f(x)f(x-1)$
   (c) $g(x) = f(x) - f(x-1)$
   (d) $g(x) = f(x-2) - 2f(x-17)$
   (e) $g(x) = [\sin(6x)]f(x)$
   (f) $g(x) = \sum_{k=x-2}^{x+4} f(k)$
   (g) $g(x) = xf(x)$
   (h) $g(x) = f(2x)$

Matlab exercises

Before starting the Matlab exercises, explore the Matlab information link on the course webpage. Review the ACS webpages with information on Matlab. Read the "Getting Started" introductions for Matlab and for the Image Processing Toolbox. Work through the "Tutorial Code Snippets."

1. Working with Images in Matlab.
   (a) Download Figure 1.14(c) from the textbook website photo gallery. One way of doing this is as follows. First, from the Unix/Linux command line, use `wget` to grab the image as follows:
   ```
   wget http://www.imageprocessingbook.com/images/bookimages/images_chapter_01/
   Fig1.14\(c\).jpg
   ```
   Note the backslashes before the parentheses. In general, you can get this URL by cutting and pasting it from your browser after locating the image you want in the photo gallery.
   (b) In Matlab, use `imread` to load in the image,
   ```
   I=imread('/home/sjb/images/Fig1.14(c).jpg');
   ```
   where in this case I have assumed that `wget` put the file into `/home/sjb/images/`. Finally, convert the image from `uint8` to `double` by typing `I=double(I)`.

(c) Familiarize yourself with the following commands: `image`, `imagesc`, `truesize`, `pixval`, `imcrop`, `getrect`, `colormap`, `colorbar`.

(d) Make an m-file script using the above commands to do the following for the image `I`. Display it as it appears in the figure in the book, using the proper aspect ratio. Interactively select a rectangle enclosing the bottom half of the middle bottle. Crop the image using that rectangle. Display the cropped image using a gray colormap and colorbar, with the maximum and minimum gray levels corresponding to the maximum and minimum brightnesses in the image.

Things to turn in:

- Printout of your m-file.
- Printouts of program output (both the full and cropped images).

2. Convolution = Polynomial Multiplication.

(a) Convolution can be used to perform polynomial multiplication. The first step is to collect the coefficients of each polynomial into a vector. For example, $2x^2-1$ would be `[2 0 -1]`. Use `conv` to compute the product of $5x^4 - 3x^2 + x + 2$ and $x^3 + 4x - 1$. Write down the vector representations of each polynomial and that of the product.

(b) Consider two vectors of length M and N. What is the length of the convolution of these two vectors? Explain what this means in terms of polynomials and their degree. Note: by "length" we mean the number of components, as returned by the `length` command in Matlab, and not the length of the vector in the sense of linear algebra (i.e. the norm).

(c) Experiment with the function `pascal` for various values of `N`. Explain how to obtain successive rows of Pascal's triangle using convolution. In other words, with what vector can one convolve the third row `[1 2 1]` to obtain the fourth row `[1 3 3 1]`, and so on? Hint: the rows of Pascal's triangle appear as antidiagonals in the output of `pascal`, e.g. `diag(fliplr(pascal(5)))'` gives the 5th row.

Things to turn in:

- Written answers to parts 2a, 2b, 2c.

3. Moving Averages.

(a) Convolution can be thought of as a moving average. The values of the convolution kernel give the weights on each pixel used in the average. Consider the kernel `h=[1 1 1]/3`. Create a random vector `x` of length 100 using `randn`. (Note: before doing this, type `randn('seed',0)` to reset the pseudo-random number generator.) Convolve `x` with `h` and make plots before and after.

(b) Given `y=conv(x,h)` with `h` as given above, write down an expression for the value `y(k)` for generic `k`. What is the qualitative effect of convolution with `h`? This kernel is known as a 3-tap boxcar lowpass filter, or simply a box filter.

(c) Set `h` equal to the fifth row of Pascal's triangle and normalize it so that it sums to 1. Plot this kernel using `stem`. This is known as a 5-tap binomial lowpass filter. Perform the convolution with `x` and produce the plots as before. Compare this result (qualitatively) to the previous result using the boxcar. In particular, why might one want to use one filter vs. the other?

(d) Set `x` equal to row 375 of Figure 3.35(a). Convolve this signal with each of the above kernels and plot the results.

Things to turn in:

- Printout of plots for part 3a.
- Written answer for part 3b.
- Printout of plots and written answer for part 3c.
- Printout of plots for part 3d.

4. Moving Differences.

   (a) When the convolution kernel has negative values, we can think of it as a moving difference. This kind of kernel is useful for edge detection and texture analysis. Consider the kernel `[1 0 -1]/2`. This is known as the centered first difference. The kernel `[1 -1]`, which is less commonly used, is known simply as a first difference. As in Exercise 3, write down an expression for `y(k)` for each of these kernels. Why do you suppose the centered version is more commonly used?

   (b) Here are two ways of writing the derivative of a function $f'(x)$ as a limit:

   $$f'(x) = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon} \quad \text{or} \quad f'(x) = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x-\epsilon)}{2\epsilon}$$

   Explain how these definitions are related to the two first difference kernels. What does ϵ represent when $f(x)$ comes from an image?

   (c) Apply the centered first difference kernel to row 238 of Figure 3.35(a) and plot the result, before and after. This time, when you plot the signals, crop the filtered signal by removing its first and last samples. This will make it the same length as the original signal and will also align it. Explain the result: what do the positions, heights, and signs of the spikes represent?

   (d) Write down the expression for the centered second difference. Compare it to the limit definition of the second derivative. Explain how to obtain this kernel from the centered first difference kernel.

Things to turn in:

- Written answers for part 4a, 4b, 4d.
- Printout of plots and written answer for part 4c.

5. Convolution can be represented using a matrix product. The Matlab function `convmtx` produces this matrix given a convolution kernel `C` and a signal length `N`. The resulting "convolution matrix" contains shifted copies of the kernel along each row.

   (a) Generate a random vector of length 8. Compute its convolution with the kernel `[1 2 1]/4` using `conv` and `convmtx`. Show each step for both methods and demonstrate that the result is the same.

   (b) The `convmtx` function assumes the signal is zero outside the specified range. Convolution with circular or "wraparound" boundary conditions can be effected using a circulant matrix instead of `convmtx`. Circulant matrices can be produced using `gallery` with the `'circul'` option. Repeat the above convolution with circular boundary conditions using this method and display the results. Hint: the first row of the circulant matrix should be `[0.5 0.25 0 0 0 0 0 0.25]`. Indicate how the results are different at the boundaries.

Things to turn in:

- Printout of m-file and its output for part 5a.
- Printout of m-file, its output, and your written answer for part 5b.

6. Sampling and Aliasing.

   (a) Consider the function $f(x) = \cos(\omega_o x)$ with $\omega_o = 2\pi k/N$. Let $N = 16$ and consider the interval $x \in [0, 16]$. Let $f(n)$ denote $f(x)$ sampled at the integers $0, 1, \ldots, 16$. In the same graph, plot $f(x)$ (solid line) and $f(n)$ (using `stem`) for $k = 0, 1, \ldots, 16$. Use a step size of 0.1 for x and 1 for n. Use `subplot` to arrange the plots in an array on a single sheet of paper.

   (b) Indicate the value of ω_o at which $f(x)$ hits the Nyquist frequency. Note that at this point we can write $f(n) = (-1)^n$.

   (c) Now look up the word "alias" in the dictionary and write down the definition. Explain why this term is used to describe $f(n)$ when ω_o exceeds the Nyquist frequency.

   Things to turn in:

   - Printout of m-file and plot for part 6a.
   - Written answer for parts 6b, 6c.