CSE190 – Image Processing – Final

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`http://www-cse.ucsd.edu/~sjb/classes/wi02/cse190`

11:30pm-2:30pm Tue. Mar. 19, 2002.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 100. In order to get full credit you must show all your work. Good luck!

1. (4 pts) In the most general case, what is the shape of a square after an affine transformation?
2. (6 pts) Suppose we define the variable `v=[-1 2]'` in our Matlab workspace. Write down the result of each of the following operations:
   (a) `v'*v`
   (b) `v.*v`
   (c) `v*v'`
3. (25 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

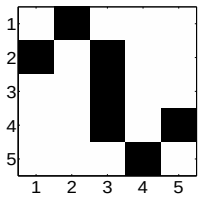

Figure 1: 5×5 binary image.

   (a) Compute the coordinates of the centroid $\mathbf{m}$.
   (b) Compute the scatter matrix C.
   (c) Find the eigenvalues λ_1 and λ_2 of C and use them to compute the aspect ratio.
   (d) Find the angle ϕ of the principal eigenvector of C. Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.
   (e) Letting $\mathbf{x}_k$ denote the original coordinates of the nonzero pixels, find the values of the rotation matrix R and translation vector $\mathbf{t}$ in the expression

$$\mathbf{x}'_k = R(\mathbf{x}_k + \mathbf{t}), \quad k = 1, 2, \ldots, 7$$

   such that the set of transformed coordinates $\mathbf{x}'_k$ for $k = 1, 2, \ldots, 7$ is centered at the origin and has its principal axis aligned with the y axis.

4. (6 pts) The slope-intercept line parameterization is given by $y_i = ax_i + b$.
   (a) What was the reason we did not use this parameterization in the Hough transform?
   (b) What parameterization did we use instead?

5. (20 pts) The expression for the 2D isotropic Gaussian is given by $h(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$.

   (a) What type of filter is $h(x,y)$: lowpass, bandpass, or highpass?

   (b) What is $H(u,v)$, the Fourier transform of $h(x,y)$? If we increase σ, what qualitative effect does it have on $H(u,v)$?

   (c) What discrete kernel is commonly used to approximate this function? Provide the entries of this kernel for the 5×5 case.

   (d) Brute-force convolution with the above 5×5 kernel requires 25 multiplications and additions to compute each pixel in the filtered image. Explain how this can be done more efficiently, and quantify the relative improvement.

6. (15 pts) This problem pertains to the measurement of optical flow.

   (a) What is the aperture problem? Write a one-sentence answer and illustrate it with a diagram.

   (b) Draw the three basic types of image neighborhoods that we considered in our discussion on optical flow, and characterize them in terms of the properties of their corresponding windowed-image second moment matrices.

7. (24 pts) The set of plots shown in Figure 2 contains 12 signals and magnitudes of their Fourier transforms. Signals 1-12 are shown in the spatial domain in the top half of the page. Each signal is of length 16. The magnitude of the DFT for each signal is shown in the bottom half of the page, labelled A-L, in random order.

   For each of the twelve signals, indicate which DFT magnitude corresponds to it. In each case, provide a brief (one-line) justification.

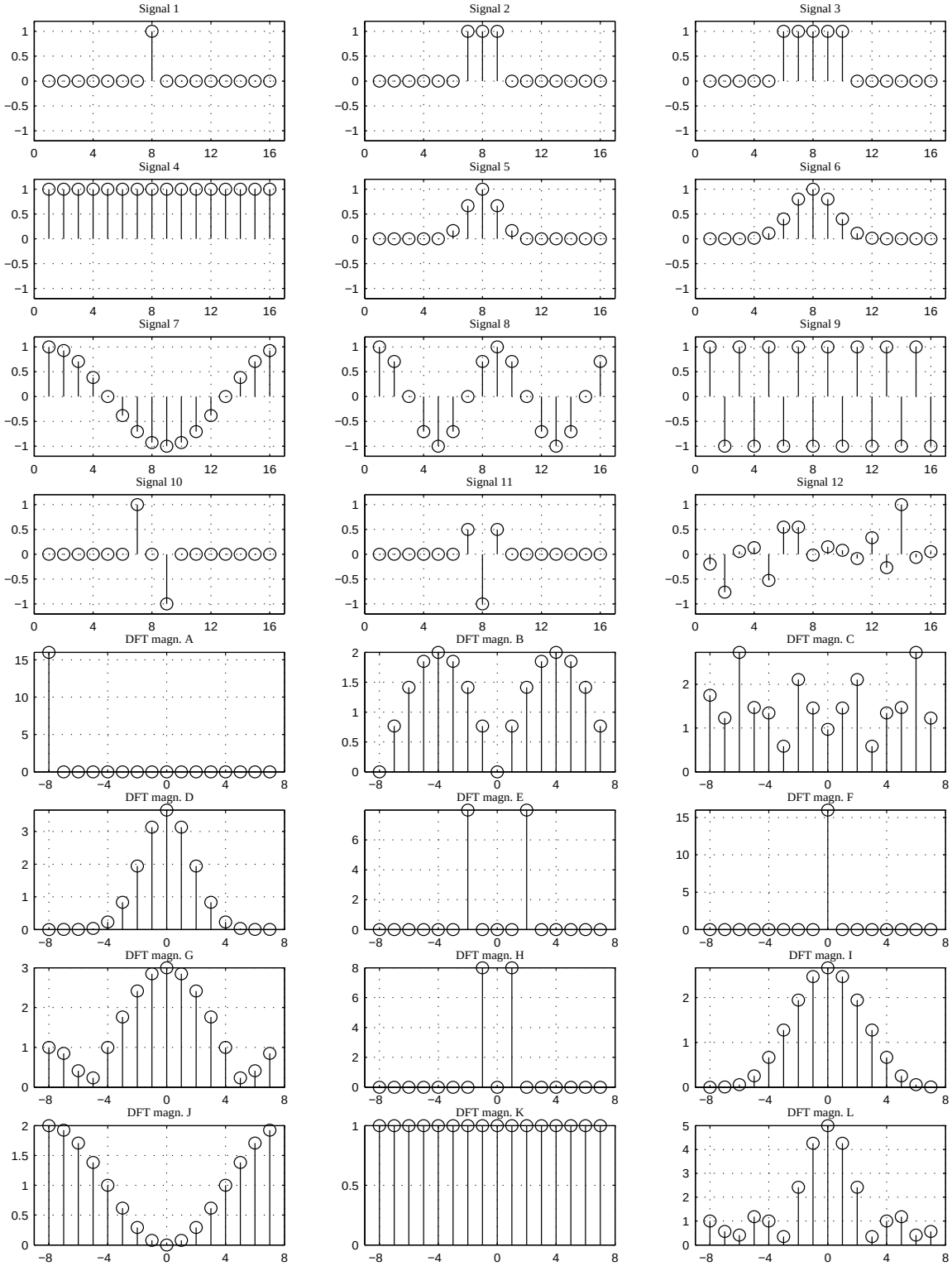

Figure 2: 12 signals and magnitudes of their Fourier transforms.