CSE 21 Midterm 2

Note: Please write your full name as it appears on the class roster and include your student id number: A01010101 for example. If you do not include both there is a chance your score will be recorded incorrectly.

Name:

Student I.D.:

Problem 1 10 points each Three girls work each Saturday night at an ice crean parlor. Girl 1 works hardest and helps 45 out of every 100 customers. Girl 2 does her share and helps 33 out of every 100 customers. The third girl helps the rest of the 100 customers. The girls mess up the orders of their customers 2 out of 100 times, 3 out of 100 times, and 5 out of 100 times, respectively.

a. Find the probability that a customer's order is messed up if they come on Saturday night to get an ice cream (only the three girls are working).

b. Suppose we know the order is messed up, what is the probability it was Girl 1 who messed it up?

Solution a. $(.45 \times .02) + (.33 \times .03) + (.22 \times .05)$.

b. $\frac{(.45 \times .02)}{a}$.

Problem 2 20 points **Solution** So the possible outcomes of length:

$$5$$

1,1,1,1,1 0r 1,1,1,1,2 or 1,1,1,1,3

$$4$$

1,1,1,2 or 1,1,1,3 or 1,2,2,2 or 1,2,1,2 or 1,2,1,3 or 1,1,2,3 or 1,1,2,1 or 1,1,2,2 or 2,1,1,1 or 2,1,1,2 or 2,1,1,3.

$$3$$

1,1,3 or 1,2,2 or 1,2,3 or 2,1,2 or 2,1,3 or 2,2,1 or 2,2,3 or 2,2,2

$$2$$

1,3 or 2,3

$$1$$

3

So using the summation for the Expected value we get:

Thus

$$f(1) = \frac{1}{3}.$$

$$f(2) = \frac{2}{9}.$$

$$f(3) = \frac{8}{27}.$$

$$f(4) = \frac{11}{81}.$$

$$f(5) = \frac{3}{243}.$$

and the expected number of tosses is

$$E(x) = f(1) + 2f(2) + 3f(3) + 4f(4) + 5f(5) = 3.15\ldots$$

Problem 3 20 points Three boys, John, Matt, and Chris, go to a bar to play darts. The probability that John hits a bullseye is $\frac{1}{10}$. Matt hits the bullseye with probability $\frac{1}{9}$, and Chris with probability $\frac{1}{8}$.

a. If each boy throws three darts find the probability that exactly one of them hits a bullseye.

Solution The solution is

$$P(chris) + P(John) + P(Matt)$$

where

$$P(chris) = 3 \times (\frac{9}{10})^3 \times (\frac{8}{9})^3 \times (\frac{7}{8})^2 \times (\frac{1}{8})^1$$

and

$$P(John) = 3 \times (\frac{1}{10}) \times (\frac{9}{10})^2 (\frac{8}{9})^3 \times (\frac{7}{8})^3$$

and

$$P(Matt) = 3 \times (\frac{1}{9}) \times (\frac{8}{9})^2 (\frac{9}{10})^3 \times (\frac{7}{8})^3$$

.

Problem 4 10 points each Let X be a continuous random variable with the following joint distribution

$$f(x) = \begin{cases} \frac{1}{6}x & 0 \leq x \leq \sqrt{12} \\ 0 & else \end{cases}$$

a. Compute $P(.2 \leq X \leq 1.2)$.

b. Also, compute the expectation, variance and standard deviation of X.

Solution a. $= \frac{7}{60}$.

b. $E(x) = 4 \times \frac{\sqrt{3}}{3}$ and $E(x^2) = 6$, so $Var(x) = \frac{2}{3}$, and the standard deviation is the square root of that.

Problem 5 A coin has $P(H) = \frac{1}{3}$.
a. If it is tossed 11 times find the probability p that the number of heads which comes up is at least 3.
b. HOw many times must the coin be tossed such that the probability of obtaining heads is at least ,5?

Solution a. So for example using the binomial formula we get that the probability that 3 heads coming up is

$$b(3, 11, \frac{1}{3}) = C(11, 3)(\frac{1}{3})^3 \times (\frac{2}{3})^8$$

similarly the probabilities of 2 heads is

$$b(2, 11, \frac{1}{3}) = C(11, 2)(\frac{1}{3})^2 \times (\frac{2}{3})^9$$

for 1 head is

$$b(1, 11, \frac{1}{3}) = C(11, 1)(\frac{1}{3})^1 \times (\frac{2}{3})^{10}$$

for 0 it is:

$$b(0, 11, \frac{1}{3}) = C(11, 0)(\frac{1}{3})^0 \times (\frac{2}{3})^{11}$$

and we seek the soluiton of at least 3

$$= 1 - (b(0, 11, \frac{1}{3}) + b(1, 11, \frac{1}{3}) + b(2, 11, \frac{1}{3})).$$

b. So we want

$$1 - (\frac{2}{3})^n > \frac{1}{2}, \rightarrow n = 2.$$