CSE 21 Midterm 1

Note: Please write your full name as it appears on the class roster and include your student id number: A01010101 for example. If you do not include both there is a chance your score will be recorded incorrectly.

Name:

Student I.D.:

Problem 1 Let $A = B \cap C \cap D$. What is $A \times A$?

Solution

$$A \times A = (B \times D) \cap (C \times B) \cap (D \times C).$$

Problem 2 a. Let $U = \{a, b, c\}$. Write down one possible partition of the power set

$$P(U).$$

b. Let $\mathcal{A} = \{\emptyset, A, A^c, U\}$ and $\mathcal{B} = \{\emptyset, B, B^c, U\}$ be algebras where A and B are subsets of U. Then what is the maximum number of elements in the set

$$\{X \cap Y : X \in \mathcal{A}, \mathcal{Y} \in \mathcal{B}\}?$$

Solution a.

$$\{\{\emptyset\}, \{\{a\}, \{b\}, \{c\}\}, \{\{a, b\}, \{a, c\}, \{c, b\}\}, \{\{a, b, c\}\}\}.$$

b. The maximum number is 10.

Problem 3 Two teams play in a basketball tournament. The first team that wins two games in a row or a total of six games wins. What is the maximum number of games that can be played?

Solution 11 games: WLWLWLWLWLW.

Problem 4 a. How many three letter words are there that can be formed from the letters in ELEVEN and have exactly two E's?

b. How many different ways can 6 boys and 5 girls be seated in a row if they are to have alternating seats?

Solution a. There are exactly 2 E's so we have L,V,N left to choose the last letter which gives $C(3,1) = 3$ choices and there are 3 possible ways to place the 2 E's. Thus we get $3 \times 3 = 9$ different words.

The pattern must be BGBGBGBGBGB so the solution is

$$P(6,6) \times P(5,5).$$

Problem 5 An ice cream parlor has 8 different flavors. We seek the number of people n needed such that the probability p that they all order different (distinct) flavors of ice cream is less then $\frac{1}{2}$, assuming they can all order only one scoop.

Solution So there are 8^n different ways in which the n people can order. Further there are $8 \times 7 \times 6 \times \ldots (8 - n + 1)$ ways in which n people can have distinct flavors. So the probability of them having distinct flavors is

$$p = \frac{8 \times 7 \times 6 \times \ldots (8 - n + 1)}{8^n}.$$

Now solve for n such that $p < \frac{1}{2}.n = 4$