CSE 20 Midterm 1

Note: Please write your full name as it appears on the class roster and include your student id number: A01010101 for example. If you do not include both there is a chance your score will be recorded incorrectly.

Name:

Student I.D.:

Problem 1 For the following statements write the contrapositive, converse and inverse.

1. If $\frac{p}{2}$ is an integer, then p is even.

2. If n is divisible by 5, then n is divisible by 10, 20 and 30.

Solution $p \to q$, so the contrapositive is $\sim q \to \sim p$, and the converse is $q \to p$, and the inverse is $\sim p \to \sim q$.

Problem 2 Given is a set of premises and a conclusion. Use the valid argument forms listed in table 1.3.1 to deduce the conclusion from the premises giving a reason for each step.

$$p \to \sim q$$

$$s \to q$$

$$p$$

$$s \vee u$$

$$\text{therefore } u.$$

Solution

1.

$$p$$

$$p \to \sim q$$

$$\text{therefore } \sim q.$$

2.

$$\sim q$$

$$\sim q \to \sim s$$

$$\text{therefore } \sim s.$$

3.

$$s \vee u$$

$$\sim s$$

$$\text{therefore } u.$$

Problem 3a Construct a Boolean expression having the given table as its truth table .

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

Problem 3b Simplify the expression obtained above and draw the corresponding circuit (it should have no more than 3 gates).

Solution So we will have

$$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R).$$

which can be simplified to

$$P \wedge (R \vee \sim Q).$$

The corresponding circuit only has 3 gates and should be easy to darw.

Problem 4 Write negations for the following two statements:

1. $\forall x \in \Re$ if $x(x+1) \neq 0$ then $x \neq 0$ and $x \neq -1$.
2. $\forall \epsilon > 0$, $\exists N_0 > 0$ such that $\forall N > N_0$ $|x - x_N| < \epsilon$.

Solution

1. $\exists x \in \Re$ such that if $x(x+1) \neq 0$ then $x = 0$ or $x = -1$.
2. $\exists \epsilon > 0$, such that $\forall N_0 > 0$ and $\forall N > N_0$ $|x - x_N| > \epsilon$.

Problem 5 Prove the following statements or give counterexamples:

1. The difference of any even and any odd integer is odd.
2. The sum of any two irrational numbers is a rational number.

Solution 1. Proof: Suppose that m is odd and n is even and $m, n \in \aleph$. Then, $m = 2r + 1$ and $n = 2s$ for some integers r and s. Now, $m + n = 2r + 2s + 1 = 2(r + s) + 1 =$ odd.

2. False, consider the counterexample $\frac{1}{4} + \frac{1}{3} = \frac{1}{12}$.

Problem 6a Represent the number $(87)_{10}$ in pental notation (base 5).

Problem 6b Represent the number $(4023)_5$ in decimal notation.

Solution $(87)_p = (322)_5$, and $(4023)_5 = 513$.