CSE 20: Discrete Mathematics

Spring Quarter 1998

Solutions for Homework Assignment #2

Section 2.1

3. d. True: $x^2 > 4 \quad \Leftrightarrow \quad x > 2$ or $x < -2 \quad \Leftrightarrow \quad |x| > 2$.

5. *Counterexample:* Let $a = 1$ or $a = -1$, and note that both

$$\frac{1-1}{1} = 0 \quad \text{and} \quad \frac{-1-1}{-1} = 2$$

are integers.

10. a. *Some acceptable answers:*
All squares are rectangles.
If a figure is a square, then that figure is a rectangle.
Every square is a rectangle.
All figures that are squares are rectangles.
Any figure that is a square is a rectangle.

b. *Some acceptable answers:*
There is a set with 16 subsets.
Some set has 16 subsets.
There is at least one set that has 16 subsets.

12. b. $\exists$ a real number x such that x is rational.

13. This problem is solved in the book.

19. This problem is solved in the book.

25. *Statement:* The product of any irrational number and any rational number is irrational.

Proposed Negation: The product of any irrational number and any rational number is rational.

The proposed negation is not correct.

Correct Negation: There are an irrational number x and a rational number y such that $x \cdot y$ is rational.

Or: Correct Negation: There are an irrational number and a rational number whose product is rational.

28. b. True.

32. $\exists$ an integer n such that n is prime and both n is not odd and $n \neq 2$.

Or: $\exists$ an integer n such that n is prime and neither is n odd nor does n equal 2.

Section 2.2

2. a. Let $n = 16$.

b. Let $n = 10^8 + 1$.

c. Let $n = 10^{10^{10}} + 1$.

8. a. There is a real number whose sum with any real number equals zero.

b. Given any real number x, there is a real number y such that $x + y \neq 0$.

12. a. $\exists$ a person x such that $\forall$ people y, x trusts y.

b. $\forall$ people x, $\exists$ a person y such that x does not trust y.

24. This problem is solved in the book.

27. *contrapositive:* $\forall$ integers n, if n is not odd and $n \neq 2$ then n is not prime.
converse: $\forall$ integers n, if n is odd or $n = 2$ then n is prime.
inverse: $\forall$ integers n, if n is not prime, then both n is not odd and $n \neq 2$.
or inverse: $\forall$ integers n, if n is not prime, then neither is n odd nor is n equal to 2.

32. If an integer is divisible by 6, then it is divisible by 3.

36. There is a person who does not have a large income and is happy.

42. a. This problem is solved in the book.

b. This statement is false: there are two distinct integers x and y such that $1/x$ and $1/y$ are both integers (let $x = 1$ and $y = -1$).

c. This statement is true: given any real number x, if $x + y = 0$ then $y = -x$ and so y is unique.

Section 2.3

2. This problem is solved in the book.

6. If a computer program is correct, then compilation of the program does not produce error messages.
Compilation of this program produces error messages.

$\therefore$ *This computer program is not correct.*

8. This problem is solved in the book.

11. invalid, converse error.

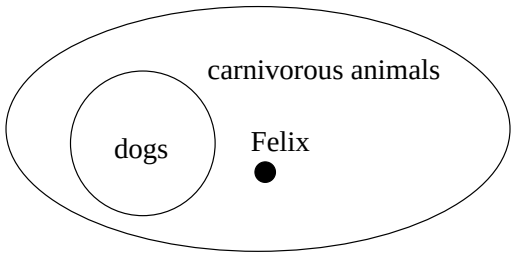
19. a. This problem is solved in the book.

b. This problem is solved in the book.

c. valid, universal modus tollens.

d. invalid, inverse error.

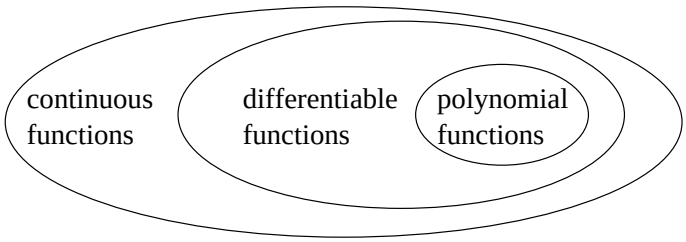
20. a.



In this case, the premises are true, but the conclusion is false.

b. The answer to a. shows that there is an argument of the given form with true premises and a false conclusion. Hence the argument form is invalid.

25. Valid. The only drawing representing the truth of the premises also represents the truth of the conclusion.



28. 2. These arguments are not arranged in regular order like the ones I'm used to.

4. If examples are not arranged in regular order like the ones I am used to, then I cannot understand them.

1. *(contrapositive form)* If I can't understand a logic example, then I grumble when I work it.

5. If I grumble at an example, then it gives me a headache.

3. *(contrapositive form)* If an example makes my head ache, then it is not easy.

$\therefore$ These arguments are not easy.