CSE 20: Discrete Mathematics

Spring Quarter 1998

Solutions for Homework Assignment #1

Section 1.1

4. a. The common form is:

   If p then q.
   If q then r.
   Therefore, if p then r.

   b. If x *equals* 0, then the guard condition for the **while** loop is false.
   If *the guard condition for the **while** loop is false*, then program execution moves to the next instruction following the loop.
   Therefore, if x equals 0, then *program execution moves to the next instruction following the loop.*

11. It is used in an inclusive sense. A team will also win if it wins two games in a row *and* a total of three games.

15.

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	F	T	F
T	T	F	F	T	T	F
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

18.

p	q	$p \vee q$	$\sim p$	$\sim q$	$\sim (p \vee q)$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	T

Since $\sim (p \vee q)$ and $\sim p \wedge \sim q$ have the same truth values, they are logically equivalent.

28. Sam does not swim on Thursdays or Kate does not play tennis on Saturdays.

34. The negation of $-4 < x < -1$ is

$$x \leq -4 \quad \text{or} \quad x \geq -1.$$

Note: x may be equal to -4 or -1.

40.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

all T's

Since all the truth values of $(\sim p \vee q) \vee (p \wedge \sim q)$ are T, $(\sim p \vee q) \vee (p \wedge \sim q)$ is a tautology.

44. $$\begin{aligned} p \wedge (\sim q \vee p) &\equiv p \wedge (p \vee \sim q) && \text{by the commutative law for } \vee \\ &\equiv p && \text{by the absorption law} \end{aligned}$$

Section 1.2

Show that the inverse of the converse of an implication is its contrapositive.
The converse of an implication $p \to q$ is defined as

$$q \to p$$

Since the inverse of $p \to q$ is $\sim p \to \sim q$, the inverse of the converse is

$$\sim q \to \sim p$$

which is exactly the definition of the contrapositive of $p \to q$.

4. If you do not fix my ceiling, then I won't pay my rent.

10.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

14. a. Recall that $p \to q \equiv \sim p \vee q$ (p.19). Using this equivalence, it is

$$\begin{aligned} p \to q \vee r &\equiv \sim p \vee (q \vee r) \\ &\equiv \sim p \vee q \vee r && \text{by the associative law for } \vee \end{aligned}$$

and

$$\begin{aligned} p \wedge \sim q \to r &\equiv \sim (p \wedge \sim q) \vee r \\ &\equiv \big(\sim p \vee \sim (\sim q)\big) \vee r && \text{by De Morgan's law} \\ &\equiv (\sim p \vee q) \vee r && \text{by double negative law} \\ &\equiv \sim p \vee q \vee r && \text{by the associative law for } \vee \end{aligned}$$

and similarly

$$
\begin{aligned}
p \wedge \sim r \to q &\equiv \sim (p \wedge \sim r) \ \vee q \\
&\equiv \ \big(\sim p \vee \sim (\sim r)\big) \ \vee q && \text{by De Morgan's law} \\
&\equiv \ (\sim p \ \vee r) \ \vee q && \text{by double negative law} \\
&\equiv \sim p \ \vee r \ \vee q && \text{by the associative law for } \vee \\
&\equiv \sim p \ \vee q \ \vee r && \text{by the commutative law for } \vee
\end{aligned}
$$

Since all three expressions are equivalent to $\sim p \ \vee q \ \vee r$, they are also equivalent to each other.

b. If n is prime and n is not odd, then n is 2.
And: If n is prime and n is not 2 then n is odd.

17. *Note:* $p \to q$ is false if and only if p is true and q is false.

a. $\sim p$ is false, hence $\sim p \to q$ is true.

b. p is true, hence $p \vee q$ is true.

c. q is false, hence $q \to p$ is true.

21.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

different truth values

Since $p \to q$ and $\sim p \to \sim q$ have different truth values in rows 2 and 3, they are not logically equivalent.

26. If Sam is not an expert sailor, he will not be allowed on Signe's racing boat.
If Sam is allowed on Signe's racing boat, then he is an expert sailor.

35. If this trianlge has two 45° angles, then it is a right triangle.

40. The statement "If compound X is boiling, then its temperature must be at least 250°F" is of the form "if p, then q", in symbols: $p \to q$.

a. This says $q \to p$, which is the converse and hence may be false.

b. This says $\sim q \to \sim p$, which is the contrapositive and hence equivalent to $p \to q$. So b. must be true.

c. This is "p only if q". By the definition on p.23, this means "if p then q", which is the initial assumption. Hence, c. must be true.

d. This is $\sim p \to \sim q$, the inverse of the assumption, hence may be false.

e. This says "q is a necessary condition for p". By definition (p.25) this means $\sim q \to \sim p$, which is the contrapositive and hence equivalent to $p \to q$. So e. is true.

f. This says "q is a sufficient condition for p". By definition (p.25) this means $q \to p$, which is the converse of $p \to q$ and hence may be false.

Note: To understand what is going on, it may be helpful to imagine a compound whose boiling point is 300°F. The assumption "If compound X is boiling, then its temperature must be at least 250°F" is true, yet a., d., and f. are false.

Section 1.3

1. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.
   It is not true that $\sqrt{2} = a/b$ for some integers a and b.
   $\therefore$ *$\sqrt{2}$ is not rational.*

5. If they were unsure of the address, then they would have telephoned.
   They did not telephone.
   $\therefore$ they were sure of the address.

9.

				premise		*conclusion*	
p	q	r	$q \wedge r$	$p \to q$	$p \to r$	$p \to q \wedge r$	
T	T	T	T	T	T	T	$\longleftarrow$ *critical row*
T	T	F	F	T	F	F	
T	F	T	F	F	T	F	
T	F	F	F	F	F	F	
F	T	T	T	T	T	T	$\longleftarrow$ *critical row*
F	T	F	F	T	T	T	$\longleftarrow$ *critical row*
F	F	T	F	T	T	T	$\longleftarrow$ *critical row*
F	F	F	F	T	T	T	$\longleftarrow$ *critical row*

The critical rows are those in which all the premises are true. In each critical row the conclusion is also true. Hence the argument form is valid.

18.

		premise		*conclusion*	
p	q	$p \vee q$	$\sim p$	q	
T	T	T	F	T	
T	F	T	F	F	
F	T	T	T	T	$\longleftarrow$ *critical row*
F	F	F	T	F	

The only critical row is the third, and in this row the conclusion is true. Hence the argument form is valid.

22. The logical form is

$$
\begin{array}{rl}
 & p \vee q \\
 & p \to r \\
\therefore & q \vee \sim r
\end{array}
$$

p	q	r	$\sim r$	$p \vee q$ (premise)	$p \to r$ (premise)	$q \vee \sim r$ (conclusion)	
T	T	T	F	T	T	T	$\longleftarrow$ *critical row*
T	T	F	T	T	F	T	
T	F	T	F	T	T	F	$\longleftarrow$ *critical row*
T	F	F	T	T	F	T	
F	T	T	F	T	T	T	$\longleftarrow$ *critical row*
F	T	F	T	T	T	T	$\longleftarrow$ *critical row*
F	F	T	F	F	T	F	
F	F	F	T	F	T	T	

In the third ro the premises are both true but the conclusion is false. So the argument is invalid.

25. The logical form is

$$\begin{array}{rl} & p \to q \\ & q \to r \\ \therefore & p \to r \end{array}$$

Hypothetical syllogism (example 1.3.7), valid.

32. If I have no idea what to write, then I can do this excercise.
I have no idea what to write.
$\therefore$ I can do this excercise `;-)`

39. Suppose Socko is telling the truth. Then Fats is also telling the truth because if Lefty killed Sharky then Muscles didn't kill Sharky. Consequently, two of the men were telling the truth, which contradicts the fact that all were lying except one.

Therefore, Socko is not telling the truth: Lefty did not kill Sharky. Hence Muscles is telling the truth and all others are lying.

It follows that Fats is lying, and so Muscles killed Sharky.

43.

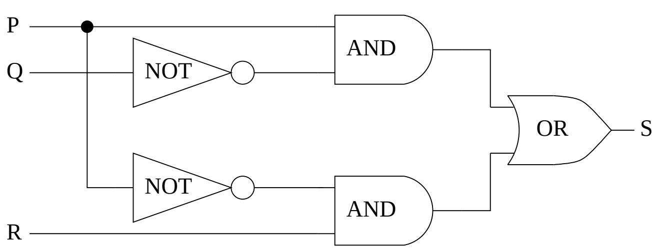
	$\sim q \vee s$	premise	
	$\sim s$	premise	
$\therefore$	$\sim q$	by disjunctive syllogism	(1)
	$p \to q$	premise	
	$\sim q$	by (1)	
$\therefore$	$\sim p$	by modus tollens	(2)
	$r \vee s$	premise	
	$\sim s$	premise	
$\therefore$	r	by disjunctive syllogism	(3)

	$\sim p$	by (2)	
	r	by (3)	
$\therefore$	$\sim p \wedge r$	by conjunctive addition	(4)
	$\sim p \wedge r \to u$	premise	
	$\sim p \wedge r$	by (4)	
$\therefore$	u	by modus ponens	(5)
	$\sim s \to \sim t$	premise	
	$\sim s$	premise	
$\therefore$	$\sim t$	by modus ponens	(6)
	$w \vee t$	premise	
	$\sim t$	by (6)	
$\therefore$	w	by disjunctive syllogism	(7)
	u	by (5)	
	w	by (7)	
$\therefore$	$u \wedge w$	by conjunctive addition	

Section 1.4

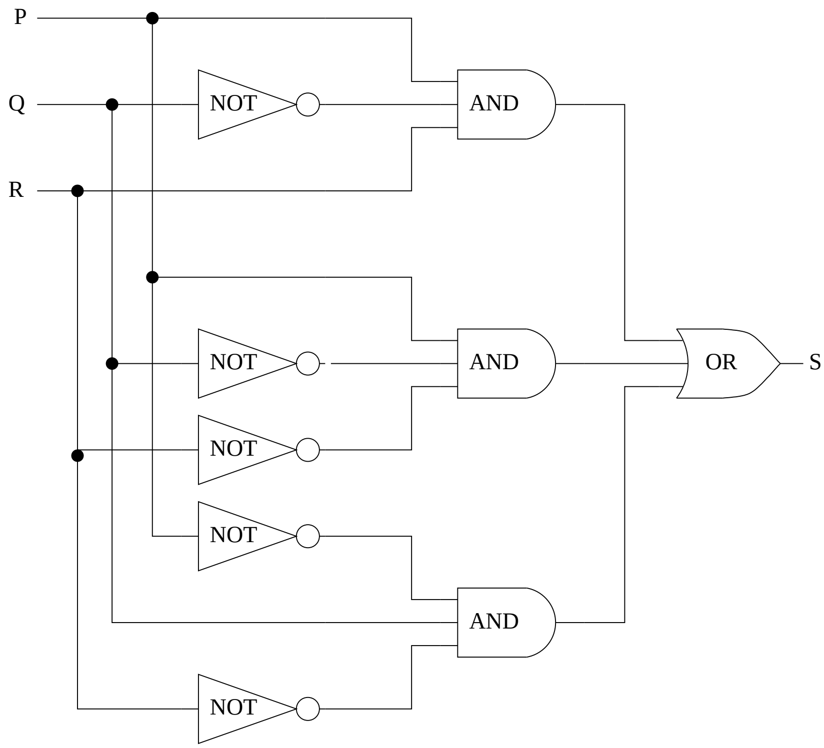
2. $R = 1$.

12. $(P \vee Q) \wedge (P \vee R)$.

17. 

19. a. $(P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R)$

b.



25. Let P, Q and R indicate the positions of the switches:

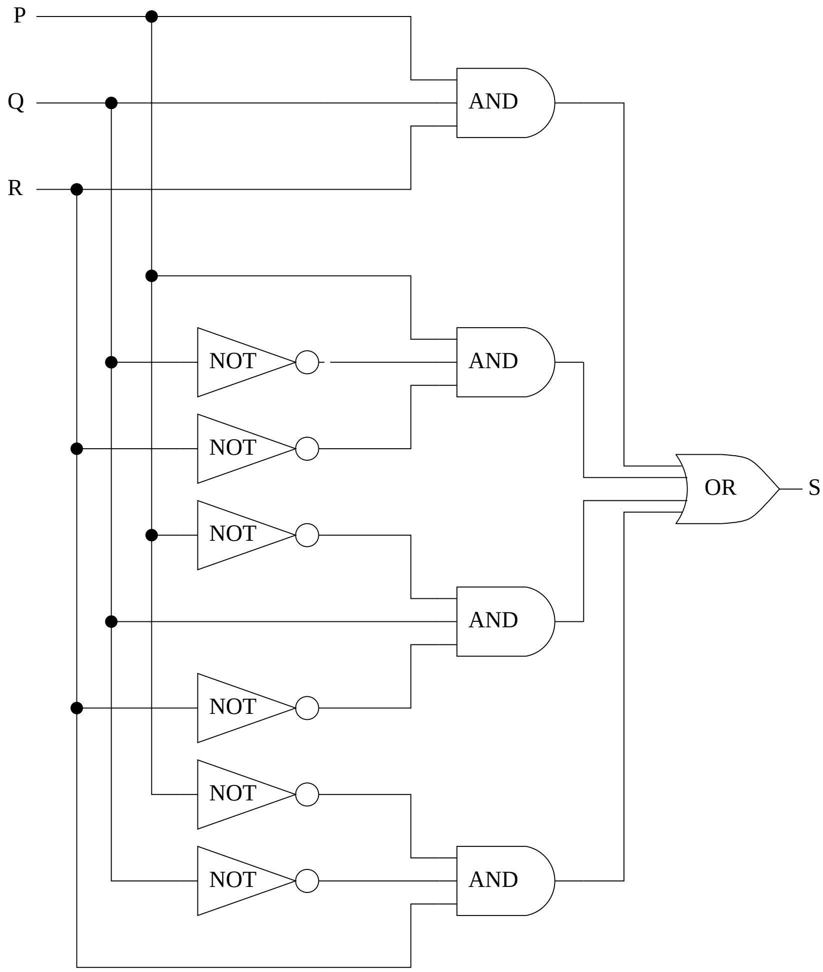
variable	switch position	0 means	1 means
P	driver's side door	door closed	door open
Q	passenger's side door	door closed	door open
R	ceiling switch	back position	front position

Let S indicate wether the light is off (0) or on (1).

Initially, $P = Q = R = 0$ and $S = 0$. Because each movement of any door or switch turns the light on if it is off and off if it is on, the complete input/output table is as follows:

input			*output*
P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

One circuit (among many) having this input/output table is the following:



Note: You will have a different answer if you define the meanings of the 1's and 0's different!

33. a.
$$\begin{aligned}(p|q)|(p|q) &\equiv \sim[(p|q)\wedge(p|q)] && \text{by definition of } |\\ &\equiv \sim(p|q) && \text{by the idempotent law for } \wedge\\ &\equiv \sim[\sim(p\wedge q)] && \text{by definition of } |\\ &\equiv p\wedge q && \text{by the double negative law}\end{aligned}$$

b.
$$\begin{aligned}p\wedge(\sim q\vee r) &\equiv \big(p\mid(\sim q\vee r)\big)\mid\big(p\mid(\sim q\vee r)\big) && \text{by part a.}\\ &\equiv \Big(p\mid\big((\sim q|\sim q)\mid(r|r)\big)\Big)\mid\Big(\ \dots\Big) && \text{by Ex. 1.4.7(b)}\\ &\equiv \Big(p\mid\big(((q|q)\mid(q|q))\mid(r|r)\big)\Big)\mid\Big(\ \dots\Big) && \text{by Ex. 1.4.7(a)}\end{aligned}$$