

Design and Analysis of Algorithms — CSE 101

Final Examination

August 1, 1998

Time: 2 hours and 30 minutes

Maximum Points: 40

NAME:

Student ID:

Answer the following questions:

1. **6 points** Describe an efficient algorithm to determine if an undirected graph $G = (V, E)$, with $n = |V|$ and $m = |E|$, is a tree. Write the pseudo-code. Determine the time complexity of your algorithm.
2. **8 points** We can use dynamic programming on a directed graph $G = (V, E)$ for speech recognition. Each edge $(u, v) \in E$ is labeled with a sound $\sigma(u, v)$ from a finite set Σ of sounds. The labeled graph is a formal model of a person speaking a restricted language. Each path in the graph starting from a distinguished vertex $v_0 \in V$ corresponds to a possible sequence of sounds produced by the model. The label of a directed path is defined to be the concatenation of the labels of the edges on that path.

Describe an efficient algorithm that, given an edge-labeled graph G with distinguished vertex v_0 and a sequence $s = (\sigma_1, \dots, \sigma_k)$ of characters from Σ , returns a path in G that begins at v_0 and has s as its label, if any such path exists. Otherwise, the algorithm should return NO-SUCH-PATH. Analyze the running time of the algorithm. Write clearly any dynamic programming formulation you may use to solve this problem.

3. **6 points** An instance of the set cover problem consists of a set X of n elements, a family F of subsets of X , and an integer k . The question is, do there exist k subsets from F whose union is X .

For example, if $X = \{1, 2, 3, 4\}$ and $F = \{\{1, 2\}, \{2, 3\}, \{4\}, \{2, 4\}\}$, there does not exist a solution for $k = 2$ but there does for $k = 3$ (for example, $\{1, 2\}, \{2, 3\}, \{4\}$).

Prove that set cover is *NP*-complete with a reduction from vertex cover.

4. **7 points** Consider the following simplification of the 0-1 knapsack problem: You have a knapsack of capacity C , and a list of items with weights w_i for $1 \leq i \leq n$. All the items have the same value. The goal is to pack the knapsack with as many items as possible. One cannot put a fraction of the item in the knapsack.

Describe an efficient algorithm for finding the optimal solution. Prove that your algorithm is correct. What is the time complexity of your algorithm?

5. **6 points** Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted order. Write an $O(\log_2 n)$ -time algorithm to find the median of all $2n$ elements in arrays X and Y . Median is an element which has exactly n elements less than or equal to it.
6. **7 points** Run Dijkstra's algorithm on the directed graph shown below to compute the shortest distances of all the nodes from the start vertex s . In each step, clearly indicate the distances and the predecessor pointers.