# Lecture 2:
# Combinational Logic

CSE 140: Components and Design Techniques for Digital Systems


Diba Mirza

Dept. of Computer Science and Engineering

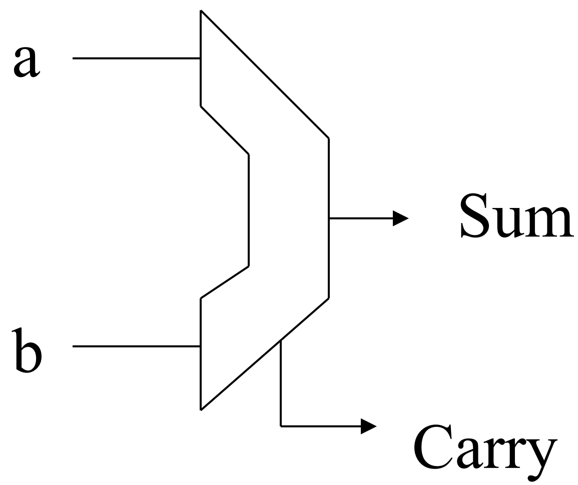
University of California, San Diego

# Outline

- What is a combinational circuit?
- Combinational Logic
  1. Scope
  2. Specification : Boolean algebra, truth tables
  3. Synthesis: circuits

# Specifying Logic Problems: Truth tables

## Example: Half Adder




Truth Table

| a | b | carry | sum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# The process of designing the circuit

Truth Table

| a | b | carry | sum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Switching Function

How?

Switching Expressions or Boolean Equation:

$$\text{Sum}(a, b) = a'b + ab'$$

$$\text{Carry}(a, b) = ab$$

# How do we get a Boolean Equation from the truth table?
# (Recall Disjunctive Normal Form in CSE 20)

Truth Table

| a | b | carry | sum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

SOP
Draw the circuit

# Towards a compact representation of the disjunctive normal form

| Min term | A | B | |
|---|---|---|---|
| | 0 | 0 | |
| | 0 | 1 | |
| | 1 | 0 | |
| | 1 | 1 | |

# Sum of Product Canonical Form

| **Minterm** | **A** | **B** | **Carry** | **Sum** |
|---|---|---|---|---|
| A'B' | 0 | 0 | 0 | 0 |
| A'B | 0 | 1 | 0 | 1 |
| AB' | 1 | 0 | 0 | 1 |
| AB | 1 | 1 | 1 | 0 |

I. sum(A,B) =

II. carry(A,B)=

# Sum of Product Canonical Form

| **A** | **B** | **Y** |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

Q: Does the following SOP canonical expression correctly express the above truth table:

$$Y(A,B)= \Sigma m(2,3)$$

A. Yes

B. No

# Product of Sum Canonical Form

| Max term | A | B | Carry | Sum |
|---|---|---|---|---|
| | 0 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 1 |
| | 1 | 0 | 0 | 1 |
| | 1 | 1 | 1 | 0 |

For the SOP expression we considered the combinations for which the output is 1

For the POS expression we will consider the combinations for which the output is 0

carry(A,B)=

Product of Sum Canonical Form is equivalent to which of the following?

A. Conjunctive normal form

B. Disjunctive normal form

# Product of Sum Canonical Form

| Max term | A | B | Carry | Sum |
|---|---|---|---|---|
| | 0 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 1 |
| | 1 | 0 | 0 | 1 |
| | 1 | 1 | 1 | 0 |

The POS expression for sum(A,B)

A.(A' +B).(A+B' )

B.A' B + AB'

C.(A+B' ).(A' +B)

D.Either A or C

E.None of the above

# Re-deriving the truth table

Truth Table

| A | B | carry | sum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

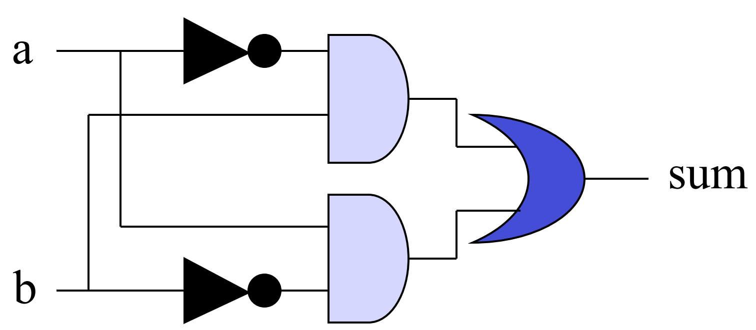
Switching Expressions:

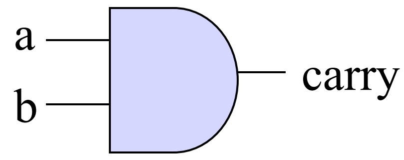
$$Sum(A,B) = A'B + AB'$$
$$Carry(A, B) = AB$$

Ex:

$$Sum(0,0) = 0'.0 + 0.0' = 0 + 0 = 0$$
$$Sum(0,1) = 0'1 + 0.1' = 1 + 0 = 1$$
$$Sum(1,1) = 1'1 + 1.1' = 0 + 0 = 0$$

## Logic circuit for half adder:



## The SOP and POS forms don’t usually give the optimal circuit or the simplest Boolean expression of the switching function

To optimize the circuit, we need to simplify the Boolean expression using:

1. Boolean Algebra axioms and theorems
2. Karnaugh Maps (K-Maps) (next lecture)

# Axioms of Boolean Algebra

1. B = 0, if B not equal to 1
2. $0' = 1$
3. $1.1 = 1$
4. $0.1 = 0$
5. $a+0=a$, $a.1=a$ Identity law
6. $a+a'=1$, $a.a'=0$ Complement law

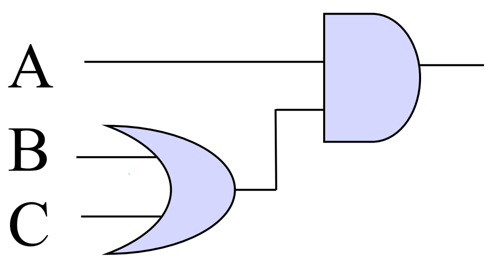
# Theorems of Boolean Algebra

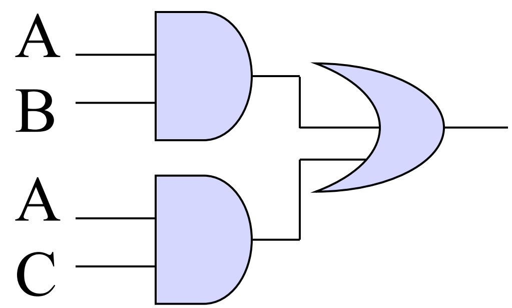
I. Commutative Law: $A + B = B + A$

$$AB = BA$$

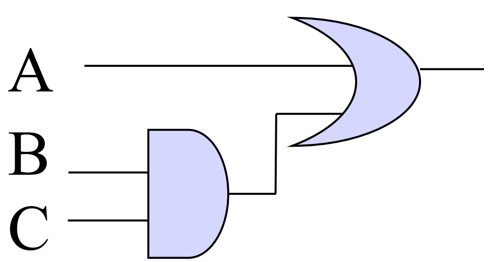
II. Distributive Law

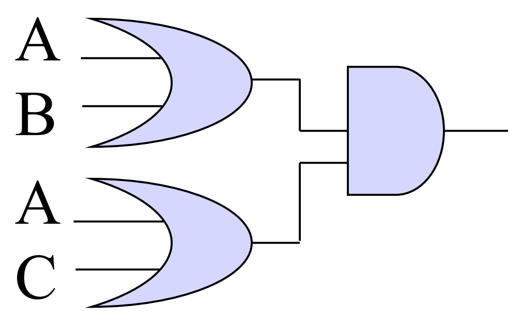
$$A(B+C) = AB + AC$$



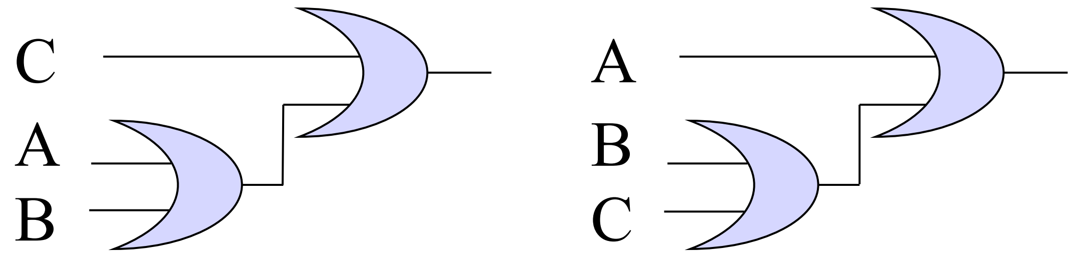

$$A+BC = (A+B)(A+C)$$



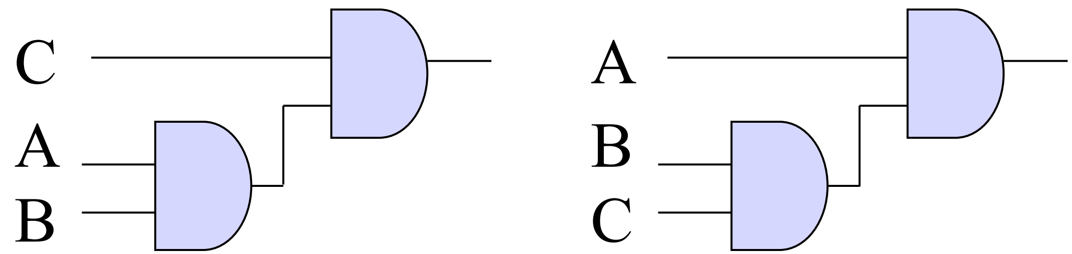



# III Associativity

$(A+B) + C = A + (B+C)$

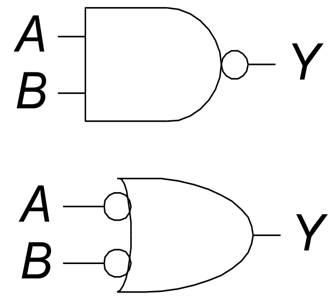



$(AB)C = A(BC)$

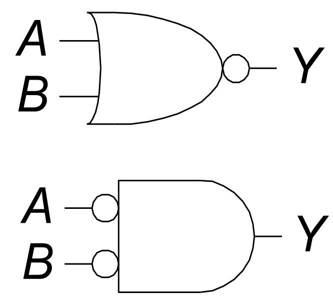


# V. DeMorgan's Theorem

- $Y = \overline{AB} = \overline{A} + \overline{B}$




- $Y = \overline{A + B} = \overline{A} \cdot \overline{B}$

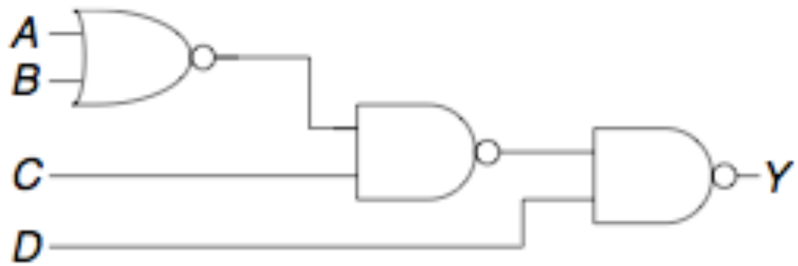


# Circuit Transformation: Bubble Pushing

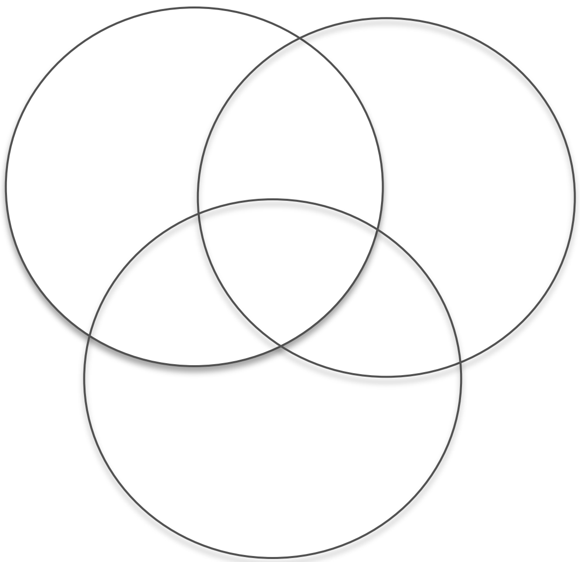
- Pushing bubbles backward (from the output) or forward (from the inputs) changes the body of the gate from AND to OR or vice versa.
- Pushing a bubble from the output back to the inputs puts bubbles on all gate inputs.

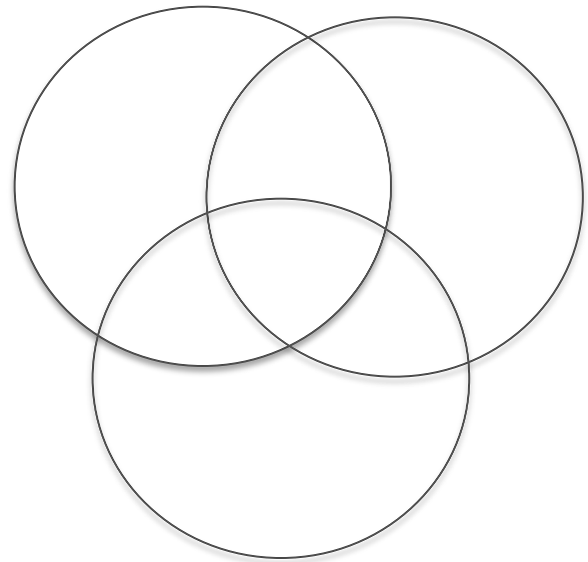

- Pushing bubbles on *all* gate inputs forward toward the output puts a bubble on the output and changes the gate body.

# Example of transforming circuits using bubble pushing



# IV: Consensus Theorem: $AB+B'C+AC = AB+B'C$





Venn Diagrams

PI Q: Which of the following is AC’ +BC+BA equal to?

A. AB+C’A

B. AC’+CB

C. BC+AB

D. None of the above

# Proof of consensus Theorem using Boolean Algebra

$AB+B'C+AC = AB+B'C$

# We are in a position to build a circuit to do n-bit Binary Addition

```
      5
  +   7
  -----
     1 2
Carry    Sum
```

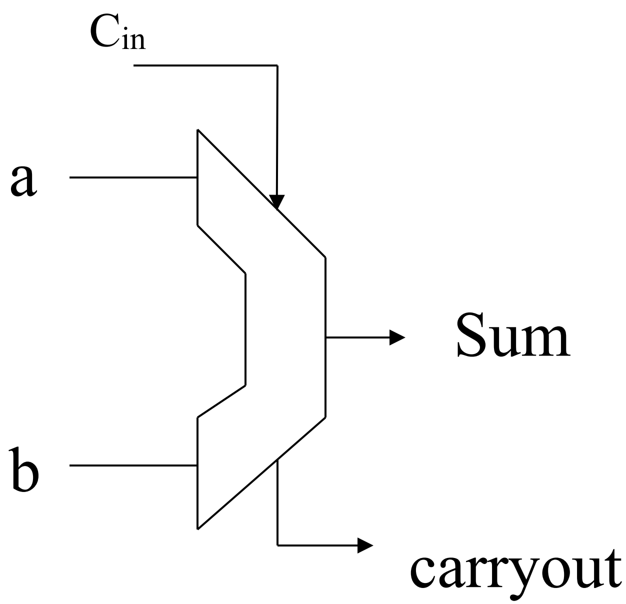
```
    1   1    1   ← Carry bits
        1   0   1       5
  +     1   1   1       7
  -----------------
     1  1   0   0      12
Carryout      Sums
```

# Binary Addition: Hardware

- Half Adder: Two inputs (a,b) and two outputs (carry, sum).

- Full Adder: Three inputs (a,b,c) and two outputs (carry, sum).

# Full Adder




Truth Table

| Id | a | b | $c_{in}$ | carryout | sum |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 1 | 1 | 1 |

# Minterm and Maxterm

| Id | a | b | $c_{in}$ | carrryout | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | | a+b+c |
| 1 | 0 | 0 | 1 | 0 | | a+b+c' |
| 2 | 0 | 1 | 0 | 0 | | a+b'+c |
| 3 | 0 | 1 | 1 | 1 | a' b c | |
| 4 | 1 | 0 | 0 | 0 | | a'+b+c |
| 5 | 1 | 0 | 1 | 1 | a b' c | |
| 6 | 1 | 1 | 0 | 1 | a b c' | |
| 7 | 1 | 1 | 1 | 1 | a b c | |

maxterm

minterm

# Minterm and Maxterm

| Id | a | b | $c_{in}$ | carryout | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | | a+b+c |
| 1 | 0 | 0 | 1 | 0 | | a+b+c' |
| 2 | 0 | 1 | 0 | 0 | | a+b'+c |
| 3 | 0 | 1 | 1 | 1 | a' b c | |
| 4 | 1 | 0 | 0 | 0 | | a'+b+c |
| 5 | 1 | 0 | 1 | 1 | a b' c | |
| 6 | 1 | 1 | 0 | 1 | a b c' | |
| 7 | 1 | 1 | 1 | 1 | a b c | |

PI Q: Is $f_1 = f_2$?

A. Yes

B. No

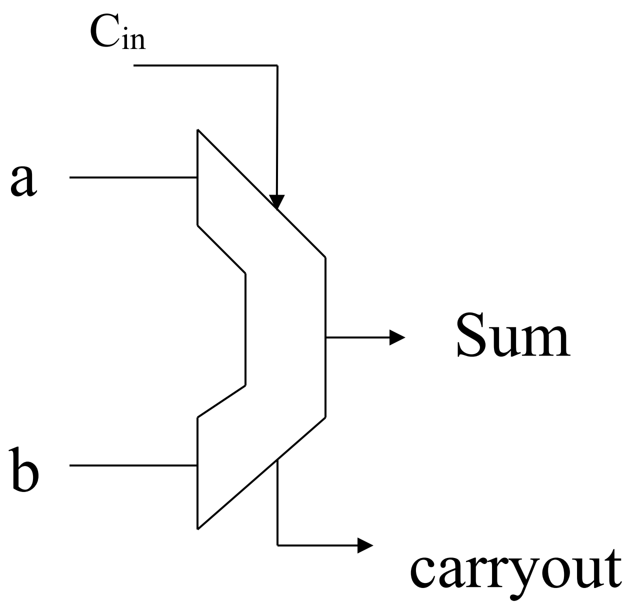
Carryout $= f_1(a,b,c) = a'bc + ab'c + abc' + abc$

$$= m_3 + m_5 + m_6 + m_7 = \Sigma m(3,5,6,7)$$

$$f_2(a,b,c) = (a+b+c)(a+b+c')(a+b'+c)(a'+b+c)$$

$$= M_0 M_1 M_2 M_4 = \Pi M(0, 1, 2, 4)$$

# Full Adder




Truth Table

| Id | a | b | $c_{in}$ | carryout | sum |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 1 | 1 | 1 |

# Circuit for full adder

Carryout (a,b,c)= $a'bc + ab'c + abc' + abc$

Sum(a,b,c) = $\Sigma m(1,2,4,7)$

# Ultimate goal…

- Optimize the circuit to be the simplest possible

- Reduction of canonical expressions

(Next lecture)