

CSE 140 Discussion

K-map Introduction

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- Recall that the expression $xyz + xy'z$ can easily be converted into xz by distributivity and complement.
- xyz and $xy'z$ are hamming adjacent.
- So if we organize our minterms such that they are hamming adjacent, we will be able to locate these larger implicants

K-map Introduction

- Meet the Karnaugh map:

x\yz	00	01	11	10
0				
1				

K-map Introduction

- Meet the Karnaugh map:
 - 3 variable K-map.

x\yz	00	01	11	10
0				
1				

K-map Introduction

- Meet the Karnaugh map:
 - 3 variable K-map.
 - Adjacent cells are hamming adjacent

x\yz	00	01	11	10
0				
1				

K-map Introduction

- Meet the Karnaugh map:
 - These are the minterms that each cell represents.

x\yz	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
1	$xy'z'$	$xy'z$	xyz	xyz'

K-map Introduction

- Meet the Karnaugh map:
 - These are the minterms that each cell represents.
 - Adjacent minterms are hamming adjacent

$x \backslash yz$	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
1	$xy'z'$	$xy'z$	xyz	xyz'

K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function

x\yz	00	01	11	10
0	0	1	1	0
1	0	0	0	0

K-map Introduction

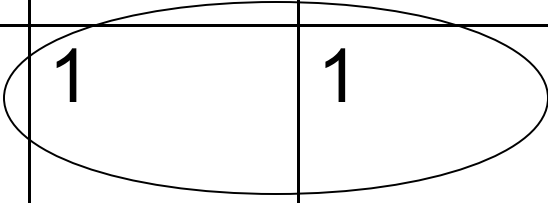
- Meet the Karnaugh map:
 - So let's say we have the following function
 - Minterms: $x'y'z$, $x'yz$

x\yz	00	01	11	10
0	0	1	1	0
1	0	0	0	0

K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function
 - Minterms: $x'y'z$, $x'yz$
 - But we can combine them into 1 implicant: $x'z$

x\yz	00	01	11	10
0	0	1	1	0
1	0	0	0	0



K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function
 - So, our function is $x'z$

x\yz	00	01	11	10
0	0	1	1	0
1	0	0	0	0

K-map Introduction

- The same thing applies to maxterms!

– Example:

$$\begin{aligned} (x + y + z')(x + y' + z') &= \\ (x + z') + (yy') &= \\ x + z' \end{aligned}$$

K-map Introduction

- Meet the Karnaugh map:
 - These are the maxterms each cell represents

$x \backslash yz$	00	01	11	10
0	$x+y+z$	$x+y+z'$	$x+y'+z'$	$x+y'+z$
1	$x'+y+z$	$x'+y+z'$	$x'+y'+z'$	$x+y'+z$

K-map Introduction

- Meet the Karnaugh map:
 - These are the maxterms each cell represents
 - Adjacent maxterms are hamming adjacent

$x \backslash yz$	00	01	11	10
0	$x+y+z$	$x+y+z'$	$x+y'+z'$	$x+y'+z$
1	$x'+y+z$	$x'+y+z'$	$x'+y'+z'$	$x+y'+z$

K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function
 - Maxterms: $(x + y + z')(x + y' + z')$

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

K-map Introduction

- Meet the Karnaugh map:
 - So let's say we have the following function
 - Maxterms: $(x + y + z')(x + y' + z')$
 - But we can combine them into 1 implicate: $x + z'$

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

K-map Practice!

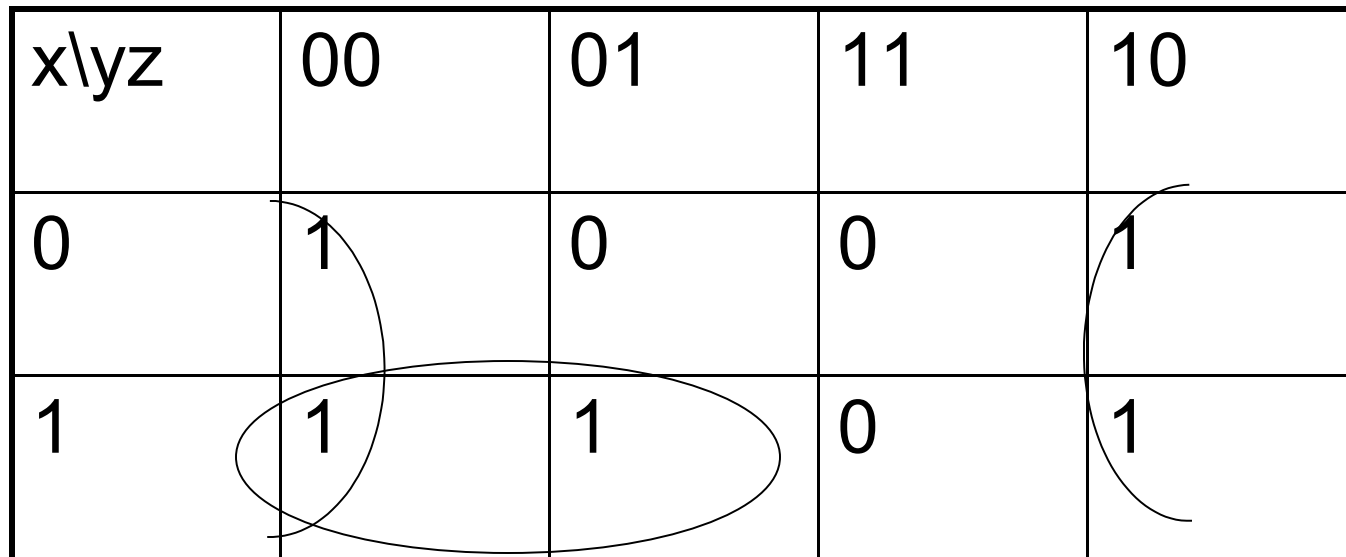
- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

K-map Practice!

- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1



The Karnaugh map shows three groups of 1s circled for simplification:

- A vertical group of two 1s in the first column (x=0, yz=00 and x=1, yz=00), representing the term $\bar{x}\bar{y}\bar{z}$.
- A horizontal group of two 1s in the second row (x=0, yz=00 and x=1, yz=00), representing the term $\bar{y}\bar{z}$.
- A vertical group of two 1s in the fourth column (x=0, yz=10 and x=1, yz=10), representing the term $\bar{x}yz$.

K-map Practice!

- Write the minimal sum-of-products form of the following:

$$z' + xy'$$

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

K-map Practice!

- Write the minimal product-of-sums form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

K-map Practice!

- Write the minimal product-of-sums form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

K-map Practice!

- Write the minimal product-of-sums form of the following:
 $(x + z')(y' + z')$

x\yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

K-map Practice!

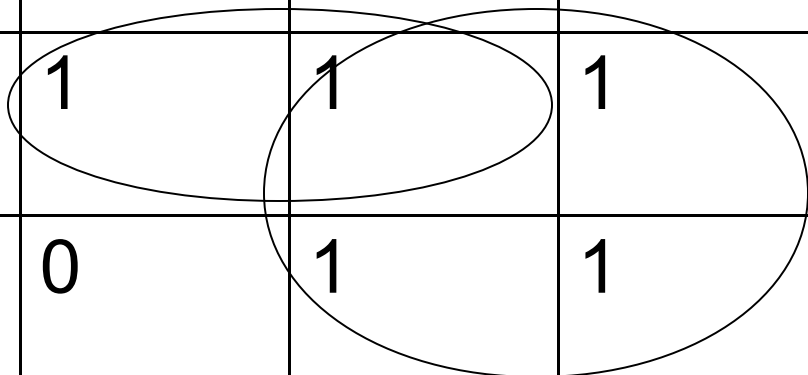
- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	1	1	0
1	0	1	1	0

K-map Practice!

- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	1	1	0
1	0	1	1	0



The Karnaugh map shows a 2x4 grid of cells. The columns are labeled 00, 01, 11, and 10. The rows are labeled 0 and 1. The cells contain the following values: (0,00)=1, (0,01)=1, (0,11)=1, (0,10)=0, (1,00)=0, (1,01)=1, (1,11)=1, (1,10)=0. Two groups of four 1s are circled: one group covers the top row (0,00), (0,01), (0,11), (0,10) and the bottom row (1,00), (1,01), (1,11), (1,10); the other group covers the first two columns (0,00), (1,00), (0,01), (1,01) and the last two columns (0,11), (1,11), (0,10), (1,10).

K-map Practice!

- Write the minimal sum-of-products form of the following:

$$x'y' + z$$

x\yz	00	01	11	10
0	1	1	1	0
1	0	1	1	0

K-map Practice!

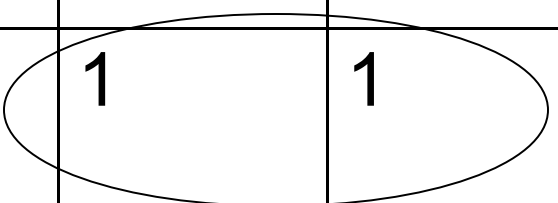
- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	0	0	1	1

K-map Practice!

- Write the minimal sum-of-products form of the following:

x\yz	00	01	11	10
0	1	0	0	1
1	0	0	1	1



K-map Practice!

- Write the minimal sum-of-products form of the following:

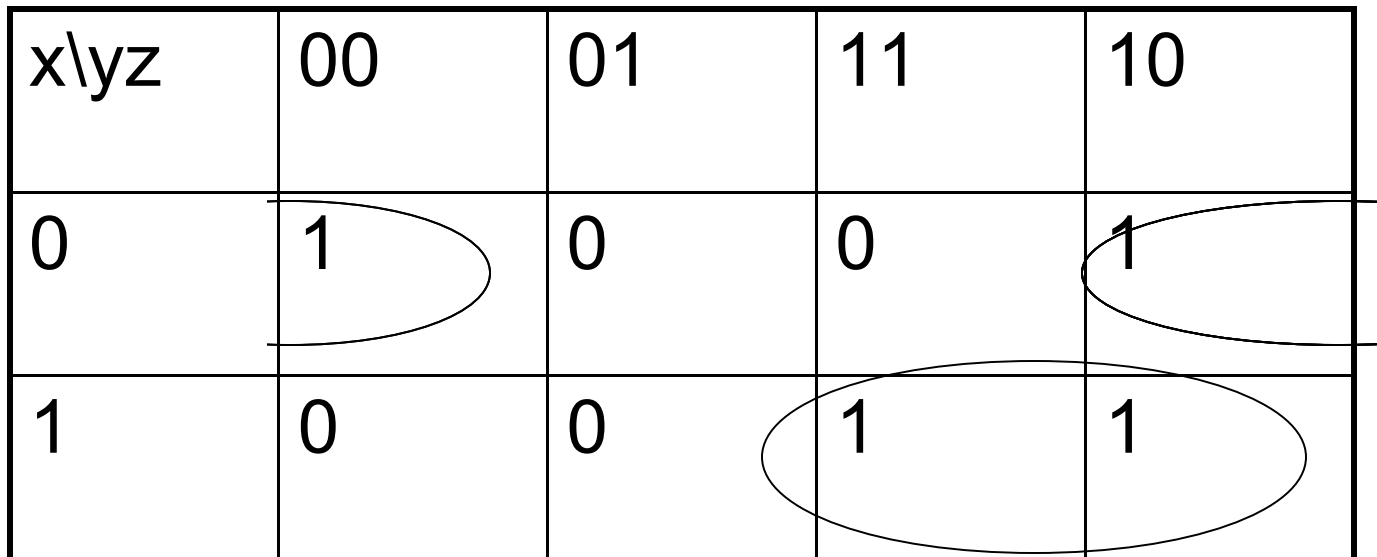
x\yz	00	01	11	10
0	1	0	0	1
1	0	0	1	1

K-map Practice!

- Write the minimal sum-of-products form of the following:

$$xy + x'z'$$

x\yz	00	01	11	10
0	1	0	0	1
1	0	0	1	1



K-map Practice!

- Write the minimal **product-of-sums** form of the following:

x\yz	00	01	11	10
0	0	1	1	0
1	0	1	1	1

K-map Practice!

- Write the minimal **product-of-sums** form of the following:

x\yz	00	01	11	10
0	0	1	1	0
1	0	1	1	1

K-map Practice!

- Write the minimal **product-of-sums** form of the following:
 $(y + z)(x + z)$

x\yz	00	01	11	10
0	0	1	1	0
1	0	1	1	1

K-map Practice!

- Write the minimal sum-of-products form of the following:

wx\yz	00	01	11	10
00	0	1	0	0
01	0	1	0	1
11	1	1	1	1
10	1	0	0	0

K-map Practice!

- Write the minimal sum-of-products form of the following:

wx\yz	00	01	11	10
00	0	1	0	0
01	0	1	0	1
11	1	1	1	1
10	1	0	0	0

K-map Practice!

- Write the minimal sum-of-products form of the following:

$$wx + w'y'z + xyz' + wy'z'$$

wx\yz	00	01	11	10
00	0	1	0	0
01	0	1	0	1
11	1	1	1	1
10	1	0	0	0

K-map Practice!

- Write the minimal product-of-sums form of the following:

wx\yz	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	1	1	1
10	0	0	0	0

K-map Practice!

- Write the minimal product-of-sums form of the following:

wx\yz	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	1	1	1
10	0	0	0	0

K-map Practice!

- Write the minimal product-of-sums form of the following:

$$(x + y)(x + z)(w' + x)(w' + y + z)$$

wx\yz	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	1	1	1
10	0	0	0	0

K-map Practice!

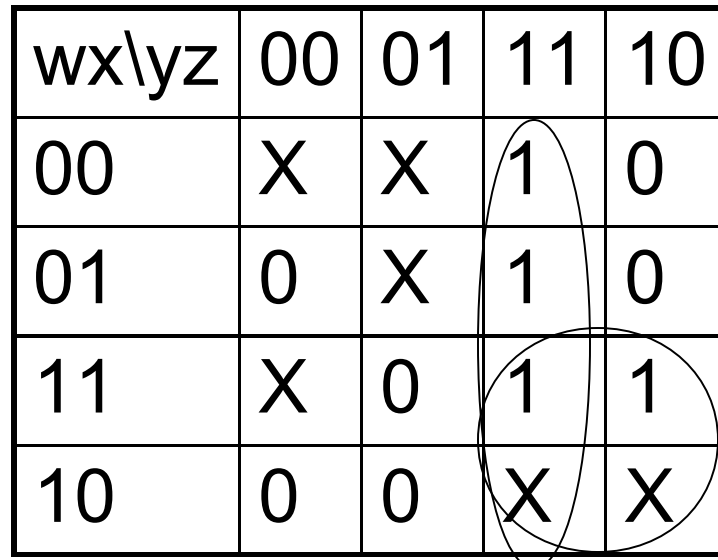
- Write the minimal sum-of-products form of the following:

wx\yz	00	01	11	10
00	X	X	1	0
01	0	X	1	0
11	X	0	1	1
10	0	0	X	X

K-map Practice!

- Write the minimal sum-of-products form of the following:

wx\yz	00	01	11	10
00	X	X	1	0
01	0	X	1	0
11	X	0	1	1
10	0	0	X	X

The K-map is a 5x5 grid. The first column contains the labels '00', '01', '11', and '10' for the rows. The first row contains the labels '00', '01', '11', and '10' for the columns. The cells contain 'X', 'X', '1', '0' for row 00; '0', 'X', '1', '0' for row 01; 'X', '0', '1', '1' for row 11; and '0', '0', 'X', 'X' for row 10. There are two hand-drawn groupings: a vertical oval enclosing the '1' cells in the third column (wx=11) for rows 00, 01, and 11; and a horizontal oval enclosing the '1' cell in the third column (wx=11, yz=11) and the '1' cell in the fourth column (wx=10, yz=11) for row 11.

K-map Practice!

- Write the minimal sum-of-products form of the following:

$$yz + wy$$

wx\yz	00	01	11	10
00	X	X	1	0
01	0	X	1	0
11	X	0	1	1
10	0	0	X	X

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:
 $x'y'$

Petrick's Method

- Let's say we have the following K-map:

$x \backslash yz$	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:
 $x'y'$, $y'z$

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:
 $x'y'$, $y'z$, xz

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:
 $x'y'$, $y'z$, xz , xy

Petrick's Method

- Let's say we have the following K-map:

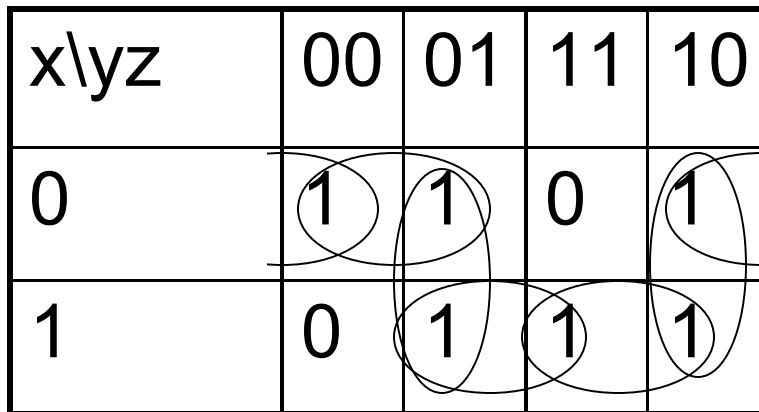
x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- First let's identify all of our implicants:
 $x'y'$, $y'z$, xz , xy , yz'

Petrick's Method

- Let's say we have the following K-map:

x\yz	00	01	11	10
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1	0	1	1	1

The Karnaugh map is a 2x4 grid. The columns are labeled 00, 01, 11, and 10. The rows are labeled 0 and 1. The cells contain the following values: (0,00)=1, (0,01)=1, (0,11)=0, (0,10)=1, (1,00)=0, (1,01)=1, (1,11)=1, (1,10)=1. There are four groupings indicated by hand-drawn circles: a horizontal group of two cells (0,00) and (0,01); a vertical group of two cells (0,01) and (1,01); a horizontal group of two cells (1,01) and (1,11); and a vertical group of two cells (0,10) and (1,10).

- First let's identify all of our implicants:
 $x'y'$, $y'z$, xz , xy , yz' , $x'z'$

Petrick's Method

- Now, let's put our implicants on a Quine-McClusky Table.

Petrick's Method

- Now, let's put our implicants on a Quine-McClusky Table.
- Let's label them as follows:

$$P0 = x'y'$$

$$P1 = y'z$$

$$P2 = xz$$

$$P3 = xy$$

$$P4 = yz'$$

$$P5 = x'z'$$

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- No row or column dominance! How do we identify the minimal cover?

Petrick's Method

- Use Petrick's Method!

Petrick's Method

- Use Petrick's Method!
 - List the implicants in product of sums form

Petrick's Method

- Use Petrick's Method!
 - List the implicants in product of sums form
 - What does this mean?

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- To cover minterm 0, we can use P0 or P5

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- To cover minterm 0, we can use P0 or P5
 $P0 + P5$

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- To cover minterm 1, we can use P0 or P1
 $(P0 + P5)(P0 + P1)$

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- To cover minterm 2, we can use P4 or P5
 $(P0 + P5)(P0 + P1)(P4 + P5)$

Petrick's Method

	0	1	2	3	4	5	6	7
P0	1	1						
P1		1				1		
P2						1		1
P3							1	1
P4			1				1	
P5	1		1					

- We do this for all covered minterms to finally get: $(P0+P5)(P0+P1)(P4+P5)(P1+P2)(P3+P4)(P2+P3)$

Petrick's Method

- Now, we simplify:

$$(P0+P5)(P0+P1)(P4+P5)(P1+P2)(P3+P4) \\ (P2+P3) =$$

Petrick's Method

- Now, we simplify:

$$(P0+P5)(P0+P1)(P4+P5)(P1+P2)(P3+P4)$$

$$(P2+P3) =$$

$$(P0+P1P5)(P4+P5P3)(P2+P1P3) =$$

Petrick's Method

- Now, we simplify:

$$(P_0+P_5)(P_0+P_1)(P_4+P_5)(P_1+P_2)(P_3+P_4)$$

$$(P_2+P_3) =$$

$$(P_0+P_1P_5)(P_4+P_5P_3)(P_2+P_1P_3) =$$

$$(P_0P_4+P_0P_3P_5+P_1P_4P_5+P_1P_3P_5)$$

$$(P_2+P_1P_3) =$$

Petrick's Method

- Now, we simplify:

$$(P_0+P_5)(P_0+P_1)(P_4+P_5)(P_1+P_2)(P_3+P_4) \\ (P_2+P_3) =$$

$$(P_0+P_1P_5)(P_4+P_5P_3)(P_2+P_1P_3) =$$

$$(P_0P_4+P_0P_3P_5+P_1P_4P_5+P_1P_3P_5)$$

$$(P_2+P_1P_3) =$$

$$P_0P_2P_4+P_0P_1P_3P_4+P_0P_2P_3P_5+P_0P_1P_3 \\ P_5+P_1P_2P_4P_5+P_1P_3P_4P_5+P_1P_2P_3P_5+ \\ P_1P_3P_5$$

Petrick's Method

- Now, we simplify:

$$(P_0+P_5)(P_0+P_1)(P_4+P_5)(P_1+P_2)(P_3+P_4)(P_2+P_3) =$$

$$(P_0+P_1P_5)(P_4+P_5P_3)(P_2+P_1P_3) =$$

$$(P_0P_4+P_0P_3P_5+P_1P_4P_5+P_1P_3P_5)$$

$$(P_2+P_1P_3) =$$

$$\mathbf{P_0P_2P_4+P_0P_1P_3P_4+P_0P_2P_3P_5+P_0P_1P_3P_5+P_1P_2P_4P_5+P_1P_3P_4P_5+P_1P_2P_3P_5+P_1P_3P_5}$$

Petrick's Method

- So, there are 2 possible minimum covers:
P0, P2, P4
P1, P3, P5
- Recall that:
 $P0 = x'y'$
 $P1 = y'z$
 $P2 = xz$
 $P3 = xy$
 $P4 = yz'$
 $P5 = x'z'$

Petrick's Method

- So our minimal covers in sum of products form are:

$$x'y' + xz + yz'$$

$$y'z + xy + x'z'$$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$x'y'$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$$x'y' + xz$$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$$x'y' + xz + yz'$$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:
 $x'y' + xz + yz'$
- Same as what we got using Petrick's Method!

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$y'z$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$$y'z + xy$$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

- Minimal Cover:

$$y'z + xy + x'z'$$

Petrick's Method

- Sanity Check:

x\yz	00	01	11	10
0	1	1	0	1
1	0	1	1	1

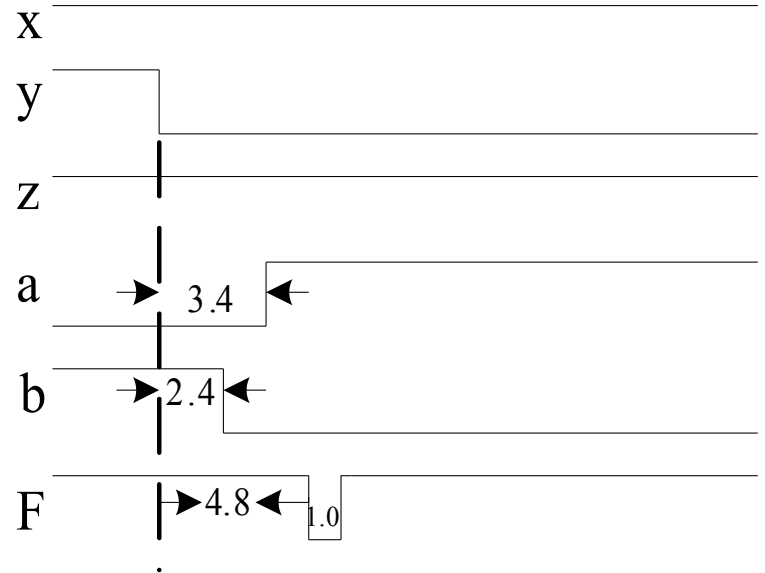
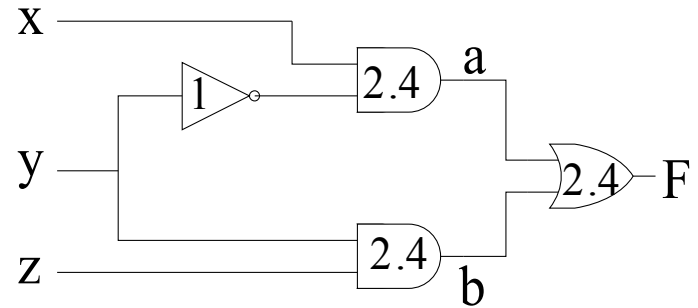
- Minimal Cover:
 $y'z + xy + x'z'$
- Same as what we got using Petrick's Method!

Static Hazards

- What are static hazards?
 - You just covered them in lab.
 - Different propagation delays result in short, unwanted delays on converging paths

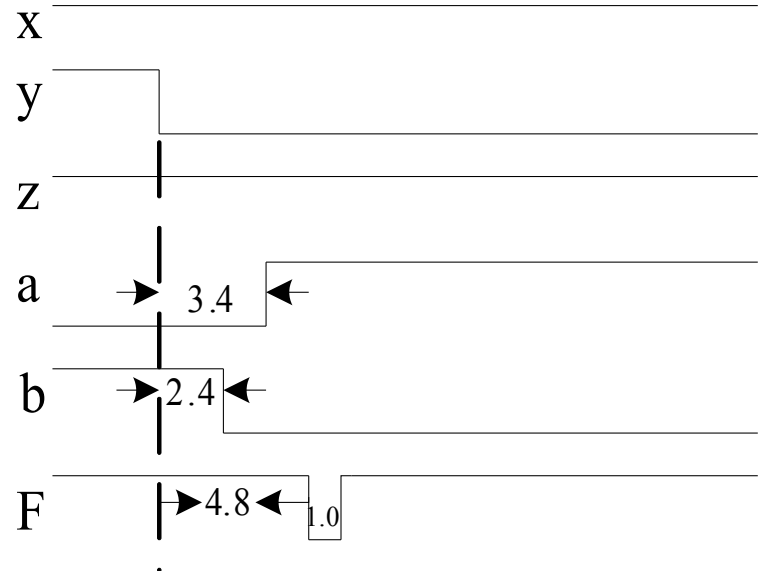
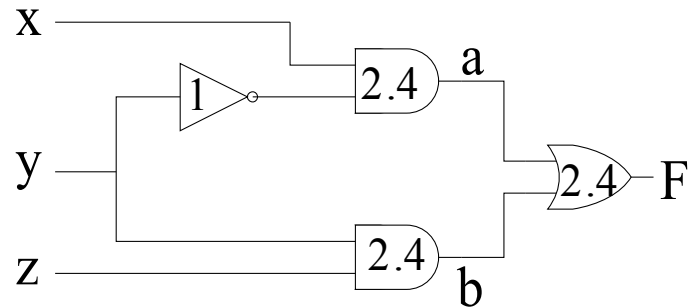
Static Hazards

- Example:



Static Hazards

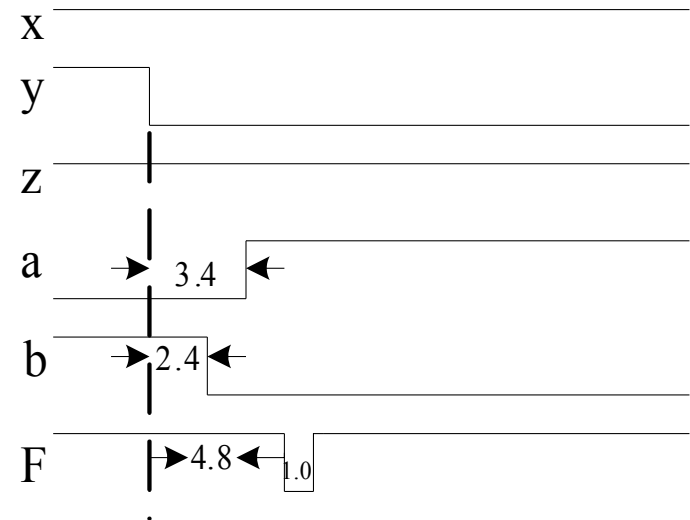
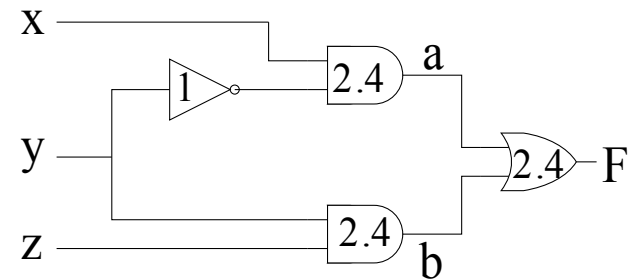
- Example:
 - How do we detect?



Static Hazards

- Example:
 - How do we detect?
 - Draw a K-map

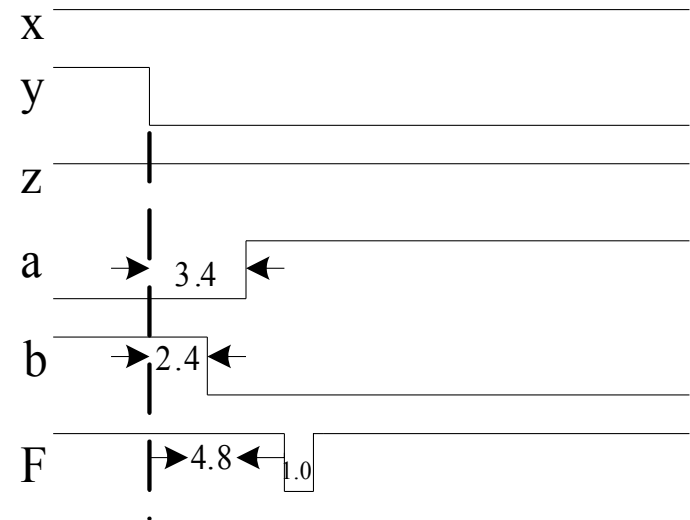
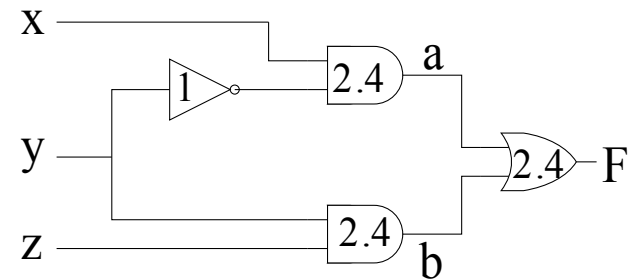
x\yz	00	01	11	10
0	0	0	1	0
1	1	1	1	0



Static Hazards

- Example:
 - How do we detect?
 - Draw a K-map
 - Circle Implicants

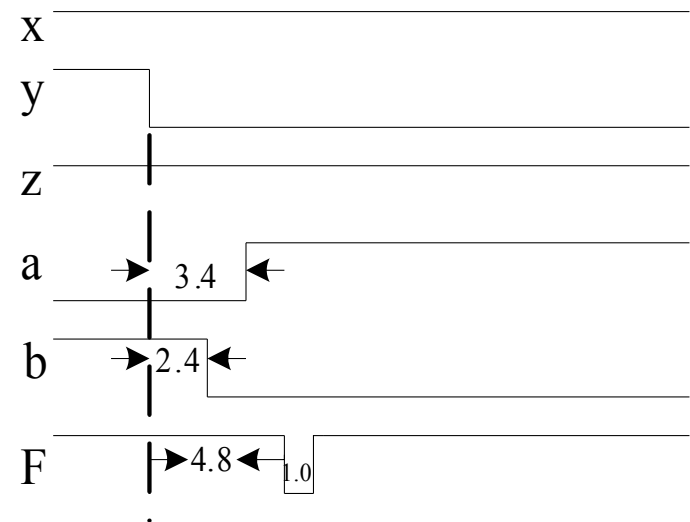
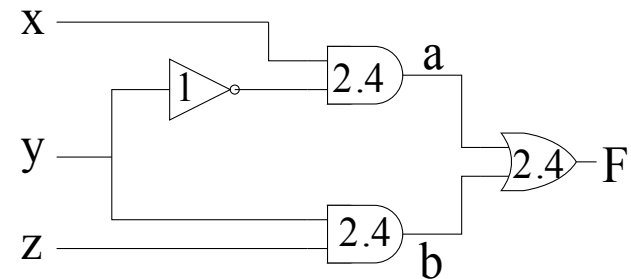
x\yz	00	01	11	10
0	0	0	1	0
1	1	1	1	0



Static Hazards

- Example:
 - How do we detect?
 - Draw a K-map
 - Circle Implicants
 - Implicants contain adjacent uncovered minterms!!!!

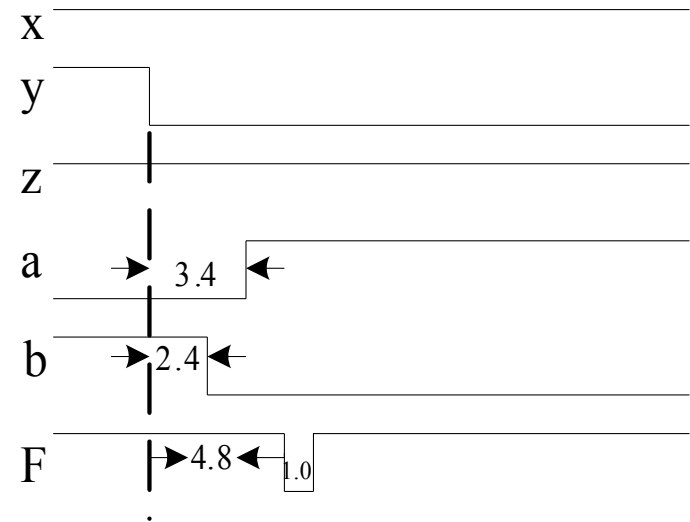
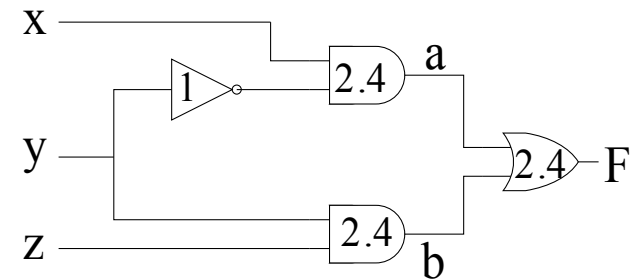
x\yz	00	01	11	10
0	0	0	1	0
1	1	1	1	0



Static Hazards

- Example:
 - Adjacent uncovered minterms result in static hazards

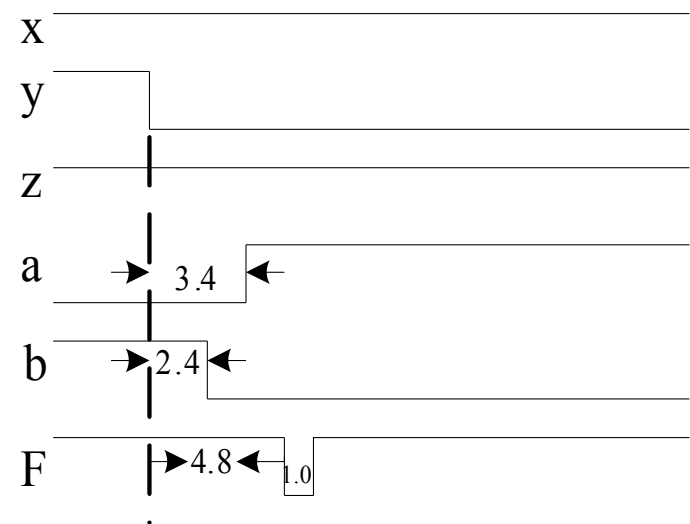
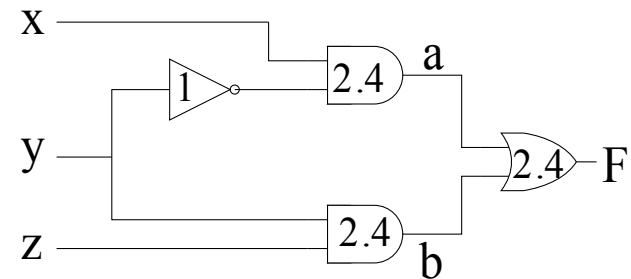
x\yz	00	01	11	10
0	0	0	1	0
1	1	1	1	0



Static Hazards

- Example:
 - Adjacent uncovered minterms result in static hazards
 - Need to cover them to eliminate hazards

x\yz	00	01	11	10
0	0	0	1	0
1	1	1	1	0



Questions??

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