

CSE 21: Math for Algorithm & System Analysis Summer II 2006

Lecture 13

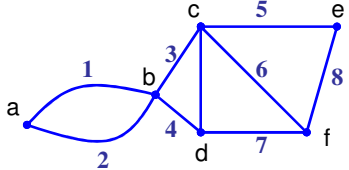
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Material covered

- Graph theory
 - A bunch of definitions
 - Graph isomorphism
 - Handshake theorem

Today's Topic

- More graph definitions
- Euler circuit
- Hamiltonian circuit

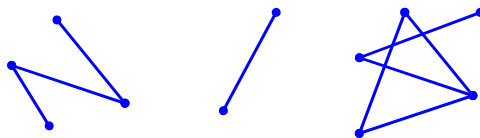
Rep edge	Rep vertex	Same start/end points	
			Definitions
			
			• Walk • Sequence of adjacent vertices and edges • Path • A walk with no repeated edges • Simple path • A path with no repeated vertices • Circuit • A path starts / ends on a same vertex • Simple circuit • A circuit with no repeated vertices except first and last

Connectedness

- *Connected vertices*: exists walk
- *Connected graph*: any two vertices connected
- *Lemma*: in a connected graph any two vertices are connected by a simple path

Connected Components

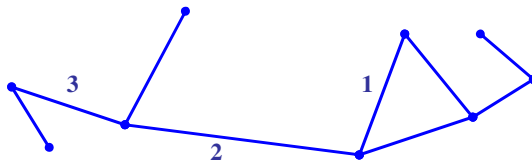
- The *connected component* of vertex v is $\{w \mid v \text{ and } w \text{ are connected}\}$



- The more connected components, the more broken up the graph is

Cut Edges

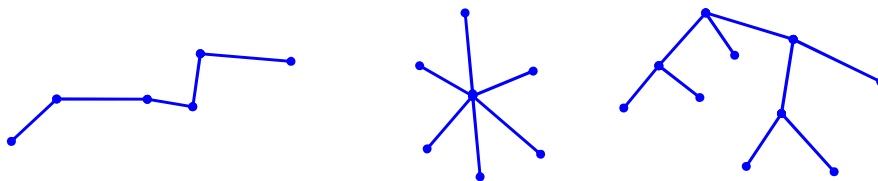
- An edge is a *cut edge* if removing it from the graph disconnects two vertices



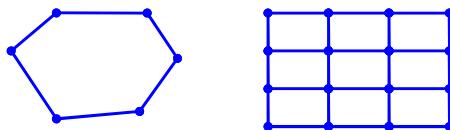
- No cut edge in a circuit
- An edge is a cut edge iff it is not part of a simple circuit

Fault-tolerant design

- Every edge is a cut edge (bad)



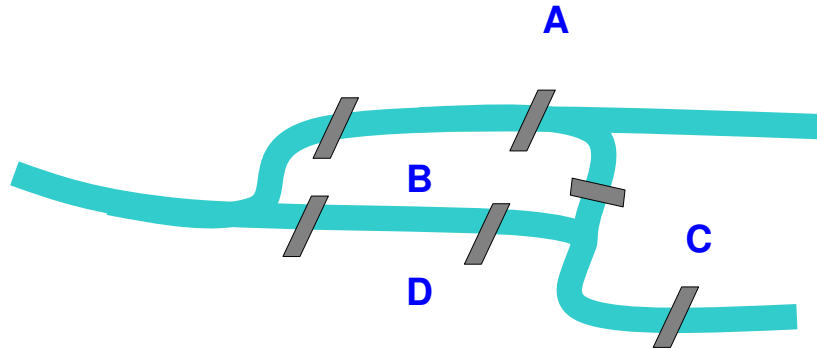
- No edge is a cut edge (good)



- Tradeoff edges for failure tolerance

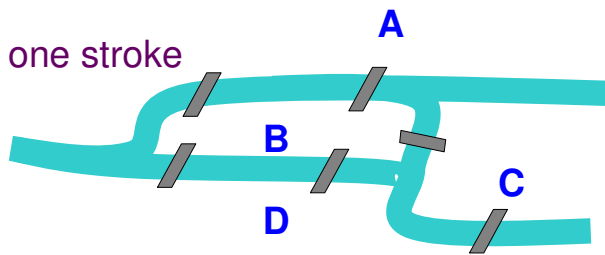
Konigsberg seven bridge problem

- Is it possible for a person to take a walk, starting and ending at the same location, crossing each bridge exactly once?



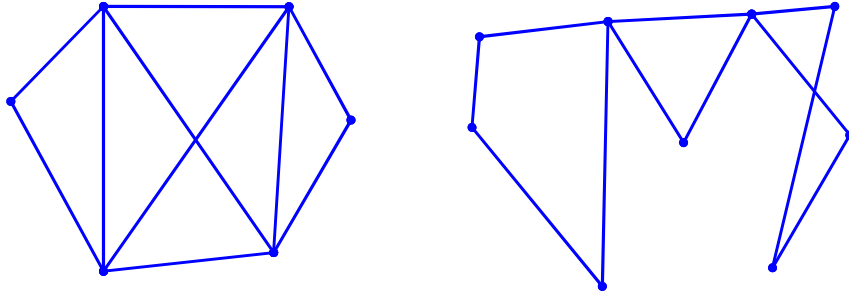
Euler Circuit

- Draw a figure in one stroke



- **Euler circuit of G :** a circuit that contains every vertex and every edge
 - Each vertex $\geq$ once
 - Each edge = once

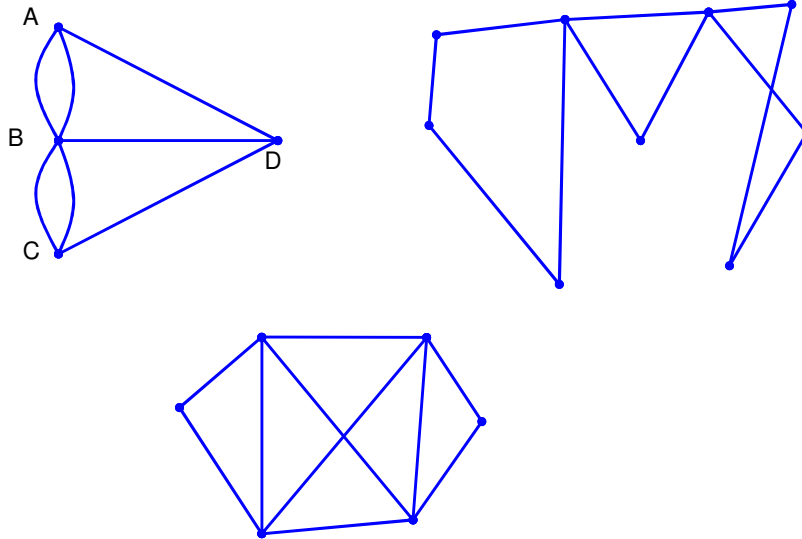
Euler circuit examples



Property of a graph with Euler circuit

- **Theorem**
 - If a graph has an Euler circuit
 - then every vertex of the graph has even degree
- **Contrapositive:**
 - If some vertex has odd degree, then the graph does not have Euler circuit

Back to the examples



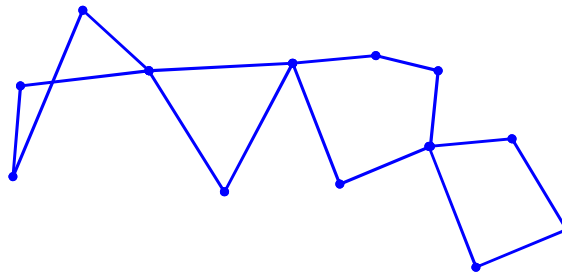
Euler circuit

- ***Is this correct?***
 - If every vertex of the graph has even degree
 - then the graph has an Euler circuit

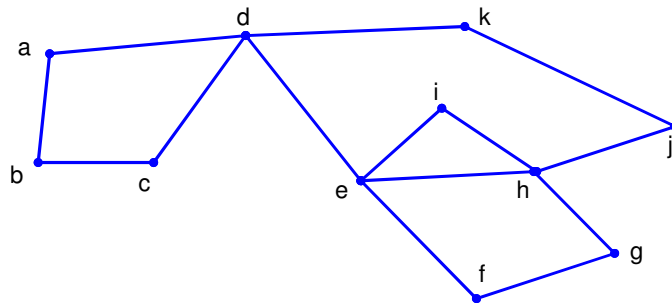
Euler circuit

- **Theorem**

- If every vertex of the graph has even degree, and
- The graph is connected
- then the graph has an Euler circuit

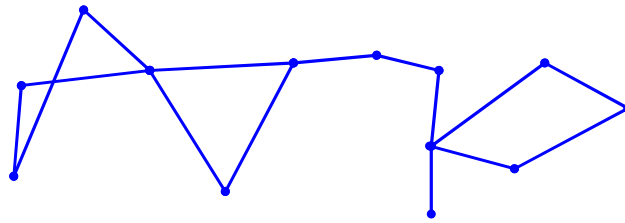


Example: Find an Euler circuit



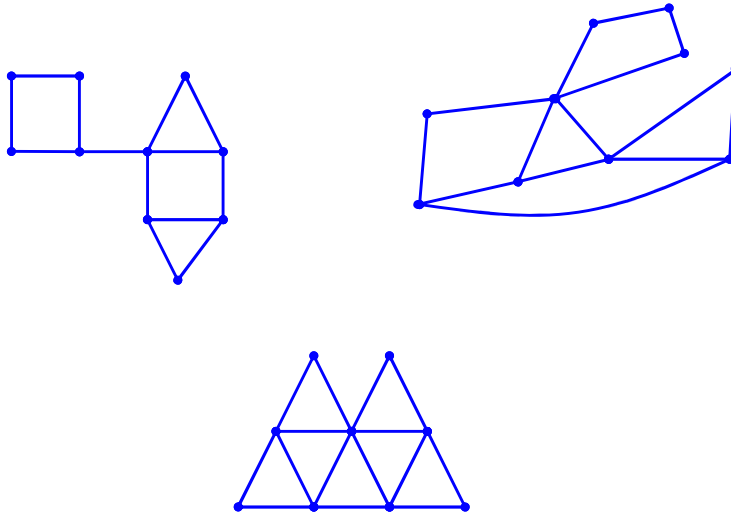
Euler circuit & Euler path

- **Theorem of Euler circuit**
 - A graph G has an Euler circuit iff G is connected and every vertex has even degree
- **Theorem of Euler path**
 - A graph G has an Euler path from distinct vertices v to w iff
 - G is connected
 - v and w have odd degree
 - all other vertices of G have even degree



Examples

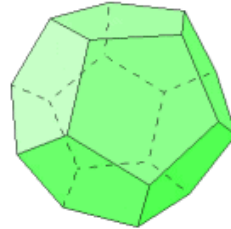
- Existing any Euler circuit / Euler path?



Hamiltonian circuit

- ***Similar to Euler circuit except***

- Must visit all vertices exactly once (rather than all edges)
- Don't need to use every edge



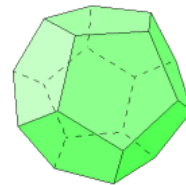
- **Example: dodecahedron**

- 12 identical pentagonal faces
- 20 vertices (representing 20 cities)
- Problem: starting from one city, tour the world by visiting each city exactly once and return to the starting city

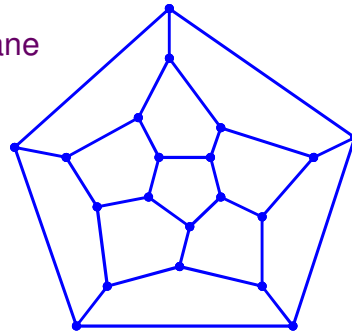
Hamiltonian circuit

- **Example: dodecahedron**

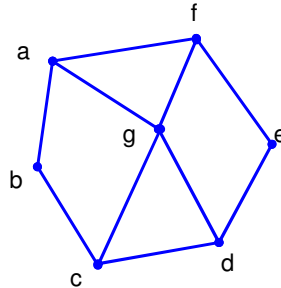
- 12 identical pentagonal faces
- 20 vertices (representing 20 cities)
- Problem: starting from one city, tour the world by visiting each city exactly once and return to the starting city



- **Stretch out a dodecahedron on a plane**

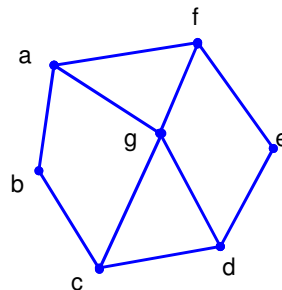
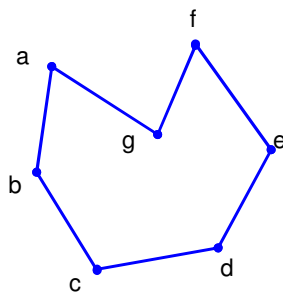


Example: Find a Hamiltonian circuit



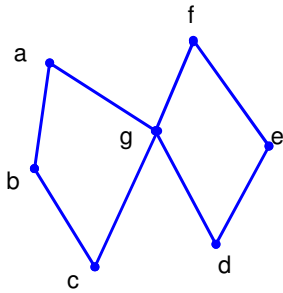
Property of a graph with Hamiltonian circuit

- ***If a graph G has a Hamiltonian circuit***
- ***Then G has a subgraph H :***
 - H contains every vertex of G
 - H is connected
 - H has the same # of edges as vertices
 - Each vertex in H has degree 2



Example: Hamiltonian circuit

- Showing the graph does not have a Hamiltonian circuit



Euler circuit vs. Hamiltonian circuit

Euler circuit

Hamiltonian circuit

Traverse
vertices

Traverse
edges

Easy to
judge?