

CSE 21: Math for Algorithm & System Analysis Summer II 2006

Lecture 12

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Material covered

- Counting
- Probability
- Recurrence
- Asymptotic notations
- Graph theory
 - Graph
 - Tree

Today's Topic

- Graph definitions
- The Handshake Theorem
- Graph Isomorphism

Graph

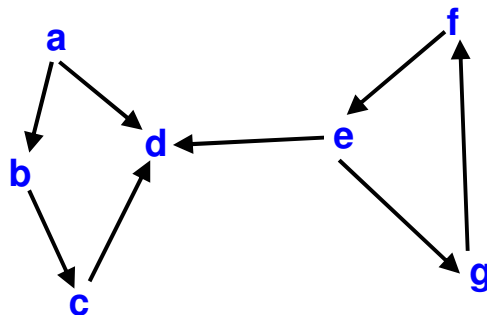
- Definition
 - **Graph** $G(V, E)$
 - V : set of **vertices**
 - E : set of **edges** (join two vertices)
 - Two vertices are **adjacent** if there is an edge between them
 - A vertex is **incident** to the edges connected to it
- Example
 - $G(V, E)$
 - $V = \{a, b, c, d\}$
 - $E = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, c\}\}$

Types of Graphs

- **Directed** graphs
 - $G(V, E)$, edge $e=(v1, v2)$
- **Undirected** graphs
 - $G(V, E)$, edge $e=\{v1, v2\}$

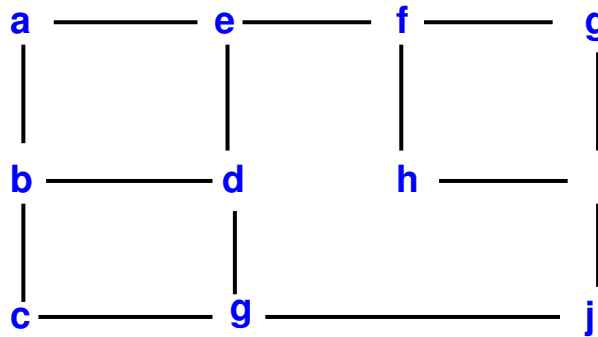
Influence Example

- Vertices are people
- Edge from i to j means i influences j
 - What is the minimal set of influencers who can spread an idea to everyone?



Street Surveillance Example

- Vertices are intersections
- Edge are streets
 - On what intersections to put police offices to patrol each edge under surveillance?

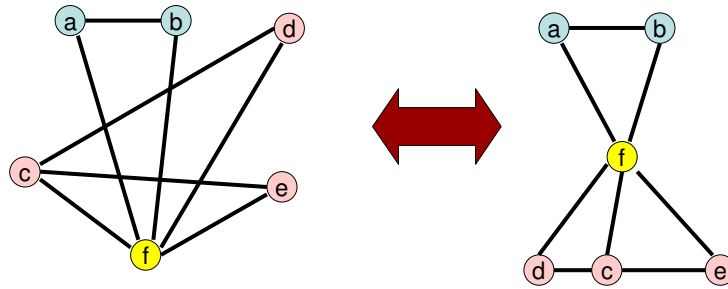


Types of Graphs

- **Loop**
 - Edge $e=\{v, v\}$
- **Parallel edges**
 - Edges $e1=\{v1, v2\}$, $e2=\{v1, v2\}$
- **Simple Graph**
 - No loop, no parallel edges

Connections not Layout

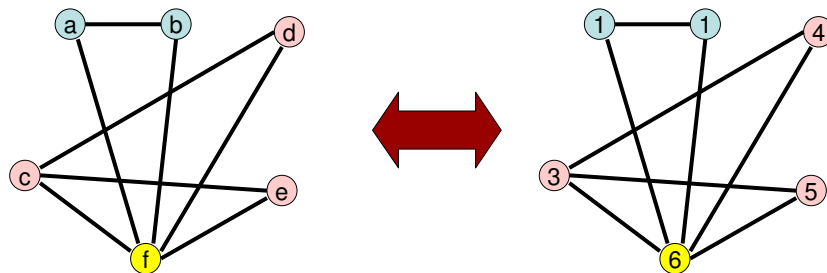
- Same graph, different layouts



- Intuition for “same graph”: can redraw G1 so that it looks like G2

Isomorphic Graphs

- Same graph, different layouts
- Same graph, different labels

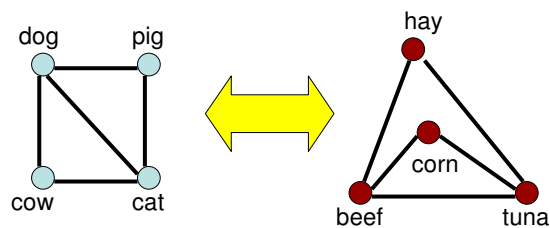


Graph Isomorphism

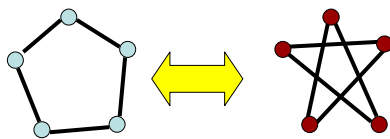
- Definition

- $G(V, E)$ is isomorphic to $G'(V', E')$:

- One-to-one mapping
 - $V \rightarrow V'$
 - $E \rightarrow E'$
 - Preserve the edge endpoint functions

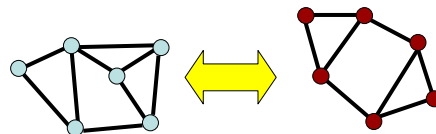


Are they isomorphic?



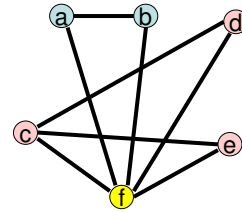
- Finding the mapping

- Not easy, may need to try all possible mappings
 - Roughly $n!$ possibilities
 - Can test for some invariants
 - # of vertices ($|V|$)
 - # of edges ($|E|$)
 - What else?



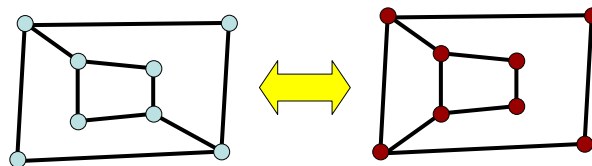
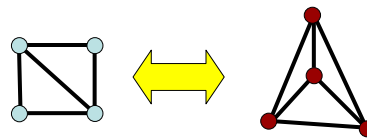
More Definitions

- **Degree** of a vertex v
 - # of edges it connects to
- $G'(V', E')$ is a **subgraph** of $G(V, E)$
 - V' is a subset of V
 - E' is a subset of E
- **Complete Graph K_n** of n vertices
 - All vertices adjacent to each other



Back to isomorphism check

- Must match (invariance)
 - # of vertices ($|V|$)
 - # of edges ($|E|$)
 - **Degrees of vertices**
 - **Subgraphs**



Possible Graph?

- Is there a graph with vertex degrees 2, 2, 1?

An impossible Graph

- $2 + 2 + 1 = \text{odd}$
- The Handshake Theorem
 - **The sum of degrees must be even**
- Proof: each edge contributes 2 to the degree sum
- Corollary:
 - The # of vertices of odd degree is even

Example about degrees

- What is the maximum possible degree in a graph of n vertices?
- Is there a graph that has n vertices all of different degrees?