

CSE 21: Math for Algorithm & System Analysis Summer II 2006

Lecture 1

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CSE 21: Math for Algo & System analysis

- **Class webpage:**
 - www.cse.ucsd.edu/classes/su06/cse21
- **Instructor:** Wenjing Rao
 - Lecture:
 - M/W/Th 6pm - 7:50pm @ WLH 2112
 - Office hour:
 - Tu 11am - 12pm @ EBU3 Room2104
- **TA:** Satoru Fukushima
 - Discussion hour:
 - Friday 6pm - 7:50pm @ WLH 2112
 - Office hour: TBD

About the course

- **Textbook**
 - *Discrete Mathematics with Applications* (3rd Edition) by Susanna S. Epp
 - Chapter covered: 6, 8, 9, 11
- **Lecture slides**
 - up.ucsd.edu
 - Classroom: CSE 21 Summer II 2006
 - Password: summer
- **WebBoard**
 - webboard.ucsd.edu
- **Gradesource**
 - Course name: cse 21 summer 2006

Grading

- **Homework (0%)**
- **Quizzes (30%)**
 - Quiz @ beginning of every lecture day.
 - Two types of quizzes
 - Individual
 - Teamwork
 - Drop the lowest 2 scores.
- **2 midterms (30%)**
 - 15% each
 - Aug 21 and Aug 31, in class.
- **1 final (40%)**
 - Sep 9 (Saturday) 7pm - 10pm @ WLH 2112

Course introduction

- **What will you be learning?**
 - Discrete ($\neq$ “discreet” !) math
 - Systematic analysis of possibilities
 - Exploration of the logic structure of a problem
 - Ingenuity
- **Why?**
 - Combinatorial reasoning required for analyzing computer systems
 - Optimization
 - Measurement
 - Probability
- **Course structure**
 - Walking through lots of problems
- **Practice! Practice! Practice!**

Topics

- Basic counting
- Discrete probability
- Recursion
- Efficiency of algorithms
- Basic graph theory

Counting: example problem 1

- There are 19 students registered for CSE21:
 - 2 sophomores
 - 7 juniors
 - 10 seniors
- How many ways are there to choose 2 students from a different level?

Counting: example problem 2

- How many different ways to create a 3-letter sequence using only **A, B, C, D, E** ?
 - With repetition
 - Without repetition
 - Without repetition and containing at least one **E**
 - With repetition and containing at least one **E**

Counting: example problem 3

- If a 5-card hand is chosen at random, what is the probability of a flush (all 5 cards of the same suit)?

Counting: example problem 4

- How many ways are there to distribute 25 different presents to 4 people (including the boss) at an office party so that the boss receives exactly twice as many presents as the second most popular person?

Collections of Objects

- Types of collections
 - Set
 - { element¹, element², ..., elementⁿ }
 - Duplicates not allowed
 - Order doesn't matter
 - List
 - (element¹, element², ..., elementⁿ)
 - Duplicates allowed
 - Order **does** matter
 - Multiset (bag)
 - Like set, but duplicates **does** allowed
 - Not often used
- Sizes
 - k-list: list with k elements (similarly: k-set and k-multiset)
 - |A|: cardinality of A (number of elements in A)

The Multiplication Rule

- If we're looking at the outcomes by making a sequence of k choices such that:
 - The *i*-th choice can be made n_i ways
 - n_i independent of previous choices
 - Each outcome can occur in only one way
 - Outcomes are all distinct
- Then the number of outcomes is $n_1 \times n_2 \times \dots \times n_k$

Multiplication Rule: Example

- There are:
 - 5 Prom King candidates
 - 6 Prom Queen candidates
- How many ways are there to pick a pair of Prom King and Queen?

Multiplication Rule “fails”

- I wish to choose one of two restaurants, and choose one item at the menu at the restaurant:
 - Restaurant A has 10 items
 - Restaurant B has 15 items

The Addition Rule

- If a finite set A can be partitioned into k distinct mutually disjoint sets: $A_1, A_2, \dots, A_k$, then:

$$|A| = |A_1| + |A_2| + \dots + |A_k|$$

The Addition Rule: Example

- There are 100 Math majors and 90 CS majors.
- How many total student?
 - No double-majors
 - 5 double-majors

Example of both Rules

- There are 19 students registered for CSE21:
 - 2 sophomores, 7 juniors, 10 seniors
 - How many ways are there to choose 2 students from a different level?
- Addition Rule:
- sophomore/junior
 - junior/senior
 - sophomore/senior

Lexicographic Ordering

“Dictionary” ordering

- **Words**
 - $A < AA < AAA < B < BAT < \dots < Z < ZY < ZZ < ZZZ < ZZZZZZ$
- **Numbers** (add leading zeros so that each number has the same number of digits)
 - $000 < 009 < 010 < 158 < 159 < 999$
- **Lists** (elements within the list need a defined ordering)
 - Define
 - $red < green < blue$
 - $A < AA < AAA$
 - $05 < 10 < 12$
 - $(red, A, 05) < (red, A, 10) < (red, A, 12) < (red, AA, 05) < \dots < (blue, AAA, 12)$

Example

- We form nonsense words using the following rules
 - Letters are from: **A, I, L, S, T**
 - Each word has 4 letters: 2 vowels and 2 consonants
 - No adjacent vowels
 - If consonants are adjacent, they are different
- What's the first word in lexicographic order?
- The last word?
- How many total words are possible?

Counting

- Come up with a way to count, so that:
 - Everything gets counted once
 - Nothing gets counted twice
- Try to break up a problem into manageable subproblems
- Find a structure in the problem that allows it to be broken into subproblems, or stages

Multiplication / Addition Rules $\Leftrightarrow$ And / Or

- Break up a problem using
 - **AND (AND THEN)**
 - **OR**
- Apply multiplication / addition rules
 - AND $\rightarrow$ Multiplication Rule
 - OR $\rightarrow$ Addition Rule

Counting Example

- How many different ways to create a 3-letter sequence using only **A, B, C, D, E** ?
 - With repetition
 - Without repetition
 - Without repetition and containing at least one **E**
 - With repetition and containing at least one **E**

Inclusion / Exclusion

- Recall: Addition Rule
 - A finite set partitioned into disjoint sets
- What if the sets are not disjoint?
- Venn Diagram

Two Sets Inclusion / Exclusion

- An internship job for CS students with certain requirements (150 CS students in total)
 - Need to know C or JAVA
 - 90 students have taken CSE5A (C)
 - 80 students have taken CSE8A (JAVA)

Inclusion / Exclusion Example

- How many arrangements of the digits 0, 1, ..., 9 are there in which the first digit is greater than 1 and the last digit is less than 8?

Three Sets Inclusion / Exclusion

- An internship job for CS students with certain requirements (150 CS students in total)
 - Need to know C or JAVA or Basic
 - 90 students have taken CSE5A (C)
 - 80 students have taken CSE8A (JAVA)
 - 40 students have self-learned Basic
 - How many students do not qualify for the job?

Three Sets Inclusion / Exclusion

Inclusion / Exclusion Example

- How many positive integers ≤ 70 are relatively prime to 70?

N-set Inclusion / Exclusion

- Let $A_1, A_2, A_3, \dots, A_n$, be sets in a universe U , denote the sum of the sizes of all k -tuple intersection of the A_i s:
 - S_k = sum of all intersections of exactly k A_i s

N-set Inclusion / Exclusion Formula

- Let $A_1, A_2, A_3, \dots, A_n$, be sets in a universe U , denote the sum of the sizes of all k -tuple intersection of the A_i s:
 - S_k = sum of all intersections of exactly k A_i s