CSE140 Class Notes 4

Quine-McCluskey Tableaux Reduction Rule

Definition 1

Two rows *a* and *b* of a reduced prime table, which cover the same minterms are said to be **interchangeable.**

Definition 2

Given two rows *a* and *b* in a reduced prime implicant table, row *a* is said to **dominate** row *b* if row *a* has checks in all the columns in which row *b* has checks and rows *a* and *b* are not interchangeable.

Definition 3

Two columns *c* and *d* of a reduced prime table, which are covered by the same prime implicants are said to be **interchangeable.**

Definition 4

Given two columns *c* and *d* in a reduced prime implicant table, column *c* is said to **dominate** column *d* if column *c* has checks in all the rows in which column *d* has checks and column *c* and *d* are not interchangeable.

Theorem 1

Let *a* and *b* be rows of a reduced prime implicant table. If *a* dominates *b* or *a* and *b* are interchangeable, there exists a minimal sum of products that does not include ***b***.

Theorem 2

Let *c* and *d* be columns of a reduced prime implicant table If *c* is dominated by *d* or *c* and *d* are interchangeable, there exists a minimal sum of products that does not include ***d***.

Example

Determine the minimal sum-of-products form for

$F(A,B,C,D,E) = \Sigma(1,2,3,5,9,10,11,18,19,20,21,23,25,26,27)$

Step 1:

Using **tabulation method,** generating all the **prime implicants** and construct a prime implicant table, as shown in Table 1.

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
P_1	C'D		X	X			X	X	X	X					X	X
P_2	BC'E					X		X						X		X
P_3	A'C'D	X		X		X		X								
P_4	A'B'D'E	X			X											
P_5	B'CD'E				X							X				
P_6	AB'CD'										X	X				
P_7	AB'DE									X			X			
P_8	AB'CE											X	X			

Table 1

Step 2:

Based on the information in Table 1, select all the **essential prime implicants**. As shown in Table 2, the EPIs (marked by a preceding *) are
C'D (P_1), BC'E (P_2), AB'CD' (P_6)

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
*P_1	C'D		X	X			X	X	X	X					X	X
*P_2	BC'E					X		X						X		X
P_3	A'C'D	X		X		X		X								
P_4	A'B'D'E	X			X											
P_5	B'CD'E				X							X				
*P_6	AB'CD'										X	X				
P_7	AB'DE									X			X			
P_8	AB'CE											X	X			

Table 2

Step 3:

Reduce the prime implicant table by crossing out the minterms already covered by the implicants selected (applying **Theorem 2**). As shown in Table 3, besides m_2, m_{10}, m_{18}, m_{25} and m_{26}, the minterm m_3, m_9, m_{11}, m_{19}, m_{21}, and m_{27} are also be covered by the P_1, P_2 or P_3. Therefore they are crossed out.

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
*P_1	C'D		X	X			X	X	X	X					X	X
*P_2	BC'E					X		X						X		X
P_3	A'C'D	X		X		X		X								
P_4	A'B'D'E	X			X											
P_5	B'CD'E				X							X				
*P_6	AB'CD'										X	X				
P_7	AB'DE									X			X			
P_8	AB'CE											X	X			

Table 3

Step 4:

In the reduced prime table, P_4 dominates P_3 and P_5, P_7 and P_8 are interchangeable. Therefore, there is a minimal cover which doe not include P_3, P_5 and P_8, as in Table 4 (applying **Theorem 1**).

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
*P_1	C’D		X	X			X	X	X	X					X	X
*P_2	BC’E					X		X						X		X
P_3	A’C’D	X		X		X		X								
P_4	A’B’D’E	X			X											
P_5	B’CD’E				X							X				
*P_6	AB’CD’										X	X				
P_7	AB’DE									X			X			
P_8	AB’CE											X	X			

Table 4

Finally the minimal cover is:

F = C’D + BC’E + A’B’D’E + AB’DE

or F = C’D + BC’E + A’B’D’E + AB’CE