University of California, San Diego
Department of Computer Science and Engineering
Course: CSE 105 − Theory of Computability

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2004 Summer Session I
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Problem Set 3

Problems 1-3 pertain to the material covered during the fourth week of class (July 19 - July 23). The remaining problems pertain to the material covered during the fifth week (July 26 - July 30).

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1. Prove that the following language is co-recognizable:

$$A = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \}$$

2. Prove that the following language is co-recognizable:

$$B = \{ \langle G, G_1, G_2 \rangle \mid G, G_1, G_2 \text{ are CFGs and } L(G) = L(G_1) \cap L(G_2) \}$$

3. Let L be a recognizable language that is not decidable. Prove that for any TM M that recognizes L there are infinitely many strings on which M does not halt.

4. Prove that the following language is undecidable:

$$A = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \}$$

5. Prove that the following language is undecidable:

$$B = \{ \langle G, G_1, G_2 \rangle \mid G, G_1, G_2 \text{ are CFGs and } L(G) = L(G_1) \cap L(G_2) \}$$

6. Prove that the following language is undecidable:

$$C = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } M_1(\varepsilon) \text{ halts and } M_2(\varepsilon) \text{ does not halt} \}$$

7. Prove that the following language is undecidable:

$$D = \{ \langle M \rangle \mid M \text{ is a TM that accepts at most one string which ends in a } \texttt{0} \}$$

8. Is the following language decidable or undecidable? Justify your answer (i.e., if E is decidable then construct a decider for it; otherwise, prove that it is undecidable).

$$E = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } L(M_1) \subseteq L(M_2) \}$$