University of California, San Diego
Department of Computer Science and Engineering
Course: CSE 105 – Theory of Computability

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Problem Set 1

This problem set is not to be handed in, and it will not be graded. But you are **strongly** encouraged to solve the problems and write down the solutions. Reading the problems and mentally solving them is not enough to prepare for Test 1; writing the solutions is an important part of the assignment. As specified on the course webpage, the grades you will receive on the exams will be based both on the correctess of your solution and on clarity of presentation. Please show your solutions to a staff member during office hours to get feedback on the quality of your write-up.

Problems 1-14 pertain to the material covered during the first week of class. The remaining problems pertain to the material covered during the second week.

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1. Give the formal description of the machine pictured in Exercise 1.16 (a) of the textbook.

2. Exercise 1.4 d, j, l.

3. Exercise 1.5 a, e.

4. Exercise 1.10 b.

5. Exercise 1.11.

6. Exercise 1.12 (a)

7. Exercise 1.13 b, e.

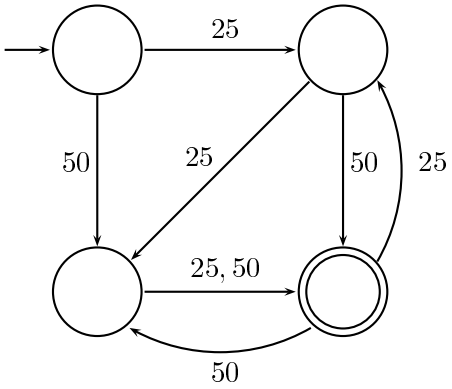
8. Problem 1.31.

9. Problem 1.32.

10. Problem 1.41.

11. Problem 1.42.

12. Consider the following DFA, where the alphabet is $\{25, 50\}$:



Assume this DFA is used to implement a vending machine that accepts only quarters and half-dollars, sells cans of Coke for \$0.75 each, and does not return change. Assume there is a sign on the machine that says "This machine does not return change."

a) Assume the vending machine is activated when a coin is inserted, and it waits for additional coins to be inserted until the customer pushes a button. The DFA processes its input as the coins are inserted, and the machine keeps track of the number of times the DFA enters an accept state – say n. When the customer pushes the button to obtain the Coke, n cans are dispensed by the machine. Give an example of an input for which the customer will be surprised to lose money.

b) What language is recognized by this DFA?

c) Give a regular expression describing the language in your answer to **b)**.

d) Now assume the vending machine waits for additional coins to be inserted only until the DFA enters an accept state. Then it immediately dispenses a can, and it does not allow more coins to be inserted until the can is dispensed. Are there inputs for which the customer will be surprised to lose money? If so, give an example.

13. Exercise 1.15 b, d, g.

14. Let Σ be an alphabet. Given a string $w = w_1 w_2 \cdots w_n$, where $w_i \in \Sigma$ for $i = 1, 2, \ldots, n$, let $even(w) = w_2 w_4 \cdots w_{n-(n \bmod 2)}$ denote the string that results from keeping only the symbols at even positions in w, and let $odd(w) = w_1 w_3 \cdots w_{n-1+(n \bmod 2)}$ denote the string that results from keeping only the symbols at odd positions in w. For example, for $\Sigma = \{0, 1\}$, we have

w	$even(w)$	$odd(w)$
0101010101	11111	00000
11111	11	111
001110110	0101	01110
0	ε	0
ε	ε	ε

The operations *even* and *odd* can be extended to languages as follows:

$$even(L) = \{\, even(w) \mid w \in L \,\} \qquad odd(L) = \{\, odd(w) \mid w \in L \,\}$$

For example, for $\Sigma = \{a, \ldots, z\}$, we have

L	$even(L)$	$odd(L)$
$\{shdjaig, ndkgyzew, sjdhl\}$	$\{hji, dgzw, jh\}$	$\{sdag, nkye, sdl\}$
$\{a, bcd\}$	$\{\varepsilon, c\}$	$\{a, bd\}$
$\{\ w \in \Sigma^* \mid \ \lvert w \rvert$ is even $\}$	Σ^*	Σ^*
Σ^*	Σ^*	Σ^*
$\emptyset$	$\emptyset$	$\emptyset$

Prove that if L is a regular language, then $even(L)$ and $odd(L)$ are both regular.

15. Exercise 1.17 c.

16. Problem 1.23 c, d.

17. Prove that the converse of the result of problem **14.** above is not necessarily true, i.e., show that there exists a language L such that both $even(L)$ and $odd(L)$ are regular, but L is not regular. [Hint: One of the languages from Problem 1.23 in the textbook will do.] Notice that once you choose a language L, you must prove that $even(L)$ and $odd(L)$ are regular (for example, by giving a regular expression or NFA for each of them, or by using closure properties of the class of regular languages) *and* prove that L is not regular (by using the pumping lemma or closure properties of the class of regular languages).

18. Exercise 2.1.

19. Exercise 2.3 a, b, ... , m.

20. Exercise 2.4 b, f.

21. Exercise 2.5 b, f.

22. Exercise 2.6 b, c.

23. Exercise 2.7 b, c. Give state diagrams also.

24. Exercise 2.8

25. Problem 2.17 a.

26. Problem 2.25

27. Prove that the class of context-free languages is closed under the concatenation operation.