Lecture 5 Overview

- Administrative issues
- Recapitulation of first week
- Regular expressions
- Equivalence of regular expressions and finite automata
- Nonregular languages

Administrative Issues

- WebBoard

  If you have been unable to log in, please send me an email.

- Class mailing list

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Let's recall what we did during the first week.

Recapitulation - First week

- Computability theory
  - ◇ What are the fundamental capabilities and limitations of "computers"?
  - ◇ What problems can "computers" solve?
  - ◇ Are there problems that "computers" intrinsically cannot solve? If so, which ones?

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  Additional Q: What other systems are equivalent?

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  - ◇ More on the class of problems solved by DFAs: Closure properties

Recapitulation - First week

Closure Properties

A **set** is *closed under an operation* if the result of applying the operation to elements in the set is also in the set.

We proved that the set (or class) of regular languages is closed under the following operations:

complement, union, intersection, concatenation, star.

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The underlined ones are called *regular operations*

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Problem Set 1 asks you to prove other closure properties of the class of regular languages.

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  - ◇ Other equivalent systems: Regular expressions

What's next?

- We will show that regular expressions are equivalent to finite automata.
- We will consider the following questions:

  Are there problems that DFAs intrinsically cannot solve? If so, which ones?

Regular Expressions

- A regular expression describes (or represents, or "generates") a set of strings.

  $L(R)$ denotes the language described by regular expression R.
- Used to describe patterns to be matched.
  - Ex.: Perl, grep, text editors such as emacs
  - Ex.: lexical analyzers in compilers
- Regular expressions are built from language "names", regular operations and parentheses.

Regular Expressions

Shorthand Notation - Examples

- 0 means $\{0\}$
  1 means $\{1\}$
  $0 \cup 1$ means $\{0\} \cup \{1\} = \{0, 1\}$
- If $\Sigma = \{a, b, c\}$ then
  Σ means $\{a\} \cup \{b\} \cup \{c\} = \{a, b, c\}$
- Concatenation operator $\cdot$ may be omitted
- Example:
  $(0 \cup 1)\Sigma = \{0a, 0b, 0c, 1a, 1b, 1c\}$

Regular Expressions - Formal Definition

Def: R is a regular expression if R is

1. s for some s in the alphabet Σ,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,
5. $(R_1 \cdot R_2)$, where R_1 and R_2 are regular expressions,

OR

6. (R_1^*), where R_1 is a regular expression

- s and ε represent languages $\{s\}$ and $\{\varepsilon\}$, respectively.
- Parenthesis may be omitted
- Precedence order: first star, then concatenation, then union

Regular Expressions - Examples

Let $\Sigma = \{0, 1\}$.

- Σ^*
  $= \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, ...\}$
  $= \{w \,|\, w$ has 0 or more ocurrences of symbols in $\Sigma\}$
- $1\Sigma^*1 \cup 0\Sigma^*0$
  $= (1\Sigma^*1) \cup (0\Sigma^*0)$
  $=$
- $\emptyset^*$
  $=$

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