Lecture 2 Overview

- Finite Automaton (DFA): definition, examples
- Regular languages
- Introduction to Nondeterministic Finite Automata (NFAs)

Finite Automaton (DFA)

- Simple computational model
- Models a computer with a very limited amount of memory

Ex. 1) Finite automaton that accepts only the key word "MAIN".

Ex. 2) Finite automaton that accepts all binary strings that end in a 0.

Ex. 3) Vending Machine

Applications of DFAs

- **Recognize patterns in strings** (e.g., key words, variable names, constants in programming languages)

  This is used in

  - Text editors (e.g., Emacs)
  - UNIX tools (e.g., grep)
  - Scripting languages (e.g., Perl)
  - Compilers: Lexical analysis phase
    Strings of symbols are partitioned into tokens of the language
    const MAX = 1000 $\longrightarrow$ [const] [MAX] [=] [1000]

Applications of DFAs

- **Keep track of limited state information** (e.g., vending machines)

  This is used in

  - Control systems (e.g., automatic door, refrigerator light control, elevator)
  - Games programming
  - Circuit design

Original Question

What problems can be solved by ... DFAs?

In order to answer this, we must define DFAs precisely.

First we will need some terminology and notation.

Terminology and Notation

- An **alphabet** is a finite set. Its members are called **symbols**.
  $\Sigma_1 = \{\mathrm{A}, \mathrm{B}, \ldots, \mathrm{Z}\}$
  $\Sigma_2 = \{0, 1\}$ (binary alphabet)
  $\Sigma_3 = \{\mathrm{n}, \mathrm{d}\}$
- A **string** is a finite sequence of symbols from an alphabet.
  MAIN is a string over Σ_1
  10101110 is a string over Σ_2
  nddddn is a string over Σ_3

Terminology and Notation

- The **length** of a string s is the number of symbols in s. It is denoted $|s|$.
  $|\text{MAIN}| = 4$
  $|10101110| = 8$
  $|\text{nddddn}| = 6$
- The **empty string**, denoted ε, is a string of length 0.
- The **concatenation** of strings x, y is the string xy.
  $x = \text{chalk}$, $y = \text{board} \Rightarrow xy = \text{chalkboard}$
  $x^2 = \text{chalkchalk}$, $xy^3 = \text{chalkboardboardboard}$
  $x\varepsilon = x$, $\varepsilon x = x$

Terminology and Notation

- A **language** is a set of strings.
  $L_1 = \{\text{MAIN}\}$
  $L_2 = \{w \mid w \text{ is a binary string ending in } 0\}$
  $L_3 = \{\text{nd}, \text{dn}, \text{dd}, \text{nnn}, \text{nnd}, \ldots\}$
- Given an alphabet Σ, Σ^* denotes the set of all strings over Σ, including ε.
  $\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, \ldots\}$
  Σ^* is a language.
  A language L over alphabet Σ is a subset of Σ^*, i.e., $L \subseteq \Sigma^*$.

Formal Definition of Finite Automaton

Def: A **finite automaton** (or **DFA**) is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of **states**
2. Σ is an alphabet
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**
4. $q_0 \in Q$ is the **start state**
5. $F \subseteq Q$ is the **set of accept (final) states**.

Note that Q, Σ and F are *sets*, δ is a *function*, and q_0 is a *state*.

Examples of DFAs

Ex. 1) DFA that "accepts" only the key word "MAIN"

$M_1 = (Q, \Sigma, \delta, q_0, F)$, where

1. $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$
2. $\Sigma = \{\mathrm{A}, \ldots, \mathrm{Z}\}$
3. $\delta(q_0, \mathrm{M}) = q_1$, $\delta(q_0, s) = q_5$ for every $s \in \Sigma \setminus \{\mathrm{M}\}$
   $\delta(q_1, \mathrm{A}) = q_2$, $\delta(q_1, s) = q_5$ for every $s \in \Sigma \setminus \{\mathrm{A}\}$
   $\vdots$
4. q_0
5. $F = \{q_4\}$

Examples of DFAs

Ex. 2) DFA that "accepts" all binary strings ending in 0

$M_2 = (Q, \Sigma, \delta, q_0, F)$, where

1. $Q = \{q_0, q_1\}$
2. $\Sigma = \{0, 1\}$
3. $\delta(q_0, 0) = q_1$
   $\delta(q_0, 1) = q_0$
   $\delta(q_1, 0) = q_1$
   $\delta(q_1, 1) = q_0$
4. q_0
5. $F = \{q_1\}$

Formal Definition of Computation for DFAs

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and $w = w_1 w_2 \cdots w_n$ a string over Σ. M **accepts** w if there is a sequence of states $r_0, r_1, \ldots, r_n \in Q$ such that

1. $r_0 = q_0$ (starts right)
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0, \ldots, n-1$ (moves right)
3. $r_n \in F$ (ends right)

Note that for all $w \in \Sigma^*$, M either accepts w or rejects w.

Formal Definition of Computation for DFAs

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. M **recognizes** a language L if $L = \{w \in \Sigma^* \mid M \text{ accepts } w\}$, i.e.,

For all $w \in \Sigma^*$:

- $w \in L \Rightarrow M$ accepts w
- $w \notin L \Rightarrow M$ rejects w

Notation: $L(M)$ denotes the language recognized by M.

Regular Languages

Def: A language L is a **regular language** if there exists a DFA M that recognizes L.

Ex. 1) $L_1 = \{\text{MAIN}\}$ is regular because M_1 recognizes it.

Ex. 2) $L_2 = \{w \mid w \text{ is a binary string ending in } 0\}$ is regular because M_2 recognizes it.

Problem: Let $\Sigma = \{0, 1\}$. Prove that
$L = \{w \mid w \text{ contains the substring } 1010\}$ is regular.

DIRECT PROOF – Proof by Construction

Regular Languages

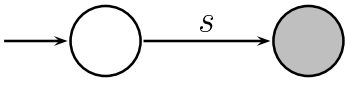
Problems

1. Prove that $L_1 = \emptyset$ is regular.
2. Let $\Sigma = \{0, 1\}$. Prove that
   $L_2 = \{w \mid w \text{ contains at least three 1s}\}$ is regular.
3. Let $\Sigma = \{0, 1\}$. Prove that $L_3 =$
   $\{w \mid w \text{ contains a 1 in the 3rd position from the end}\}$
   is regular.

Nondeterministic Finite Automaton (NFA)

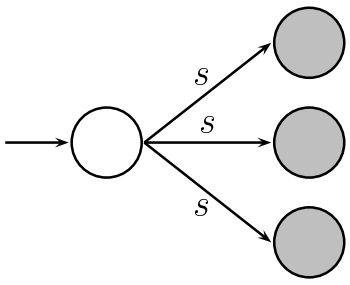
Nondeterminism allows several possible next states at every step.

Determinism:



one next state per s

Nondeterminism:



set of next states per s