Administrative Issues

- Today is the last day to request a regrade!
- Deadline to submit TRUE/FALSE and/or multiple choice questions for the final exam is W, July 28.
- Review for Test 2 on F, July 23, 11:00 AM – 12:20 PM in HSS 1330

Lecture 15 Overview

- TRUE/FALSE questions about decidability and recognizability
- Decidable Problems

TRUE/FALSE Questions

1) If there is a TM that does not halt when given input a string in a language L then L is not decidable.

FALSE.

Let $L = \Sigma^*$. Then L is decidable (A TM that always accepts decides it), but the following TM does not halt when given input a string in L:

$M =$ On input a string w
    While $(0 = 0)$ write `0` and move R

TRUE/FALSE Questions

2) If there is a TM that does not halt when given input a string in a language L then L is not recognizable.

FALSE.

The same example used for question **1)** justifies this.

TRUE/FALSE Questions

3) If L is regular then L is decidable.

TRUE.

Let L be a regular language. Then there is a DFA D that recognizes L. The following TM decides L:

$M =$ On input a string w
Simulate D on input w
If the simulation ends in an accept state, then
accept
else *reject* EndIf

TRUE/FALSE Questions

4) If L is a CFL then L is decidable.

TRUE.

Let L be a CFL. Then there is a PDA P that recognizes L. The following TM decides L: NO!

$M =$ On input a string w
Simulate P on input w
If the simulation ends in an accept state, then
accept
else *reject* EndIf

TRUE/FALSE Questions

Let L be a CFL. Then there is a PDA P that recognizes L. The following TM decides L: NO!

$M =$ On input a string w
  Simulate P on input w
  If the simulation ends in an accept state, then
    accept
  else *reject* EndIf

Some branches of P's computation might not halt. For example, some branch may push symbols onto the stack forever. The TM M would not be a decider.

TRUE/FALSE Questions

4) If L is a CFL then L is decidable.

The statement is TRUE, but we have to use a different argument to justify it.

TRUE/FALSE Questions

5) If L is finite then L is decidable.

TRUE.

Let $L = \{w_1, \ldots, w_n\}$. The following TM decides L:

$M =$ On input a string w
If $w = w_1$ or $\cdots$ or $w = w_n$ then *accept*
else *reject* EndIf

Alternative justification:

If L is finite then L is regular. By **3)**, L is decidable.

TRUE/FALSE Questions

6) If L is infinite then L is undecidable.

FALSE

Let $L = \Sigma^*$. Then L is infinite and it is decidable.

TRUE/FALSE Questions

7) If there is a TM that halts when given input a particular string in a language L then L is decidable.

FALSE

Let L be an undecidable language (we know such languages exist). The following TM halts when given input a particular string in L:

$M =$ On input a string w
accept

TRUE/FALSE Questions

8) If there is a TM that halts when given input any string in a language L then L is decidable.

FALSE

The same example used for question **7)** justifies this.

TRUE/FALSE Questions

9) If there is a TM that rejects some string in a language L then L is not recognizable.

FALSE

Let $L = \Sigma^*$. Then L is recognizable, but the following TM rejects a string in L.

$M =$ On input a string w
reject

TRUE/FALSE Questions

10) If L is recognizable then $\overline{L}$ is recognizable.

FALSE

There are languages L such that L is recognizable but $\overline{L}$ is not recognizable. We will discuss examples of such languages next week.

TRUE/FALSE Questions

11) If L is recognizable and $\overline{L}$ is recognizable then L is decidable.

TRUE

Let L be a recognizable language such that $\overline{L}$ is recognizable. Then there are TMs M and $\overline{M}$ that recognize L and $\overline{L}$, respectively. The following TM decides L:

TRUE/FALSE Questions

11) If L is recognizable and $\overline{L}$ is recognizable then L is decidable.

TRUE

The following TM decides L:

$D =$ On input a string w
Run $M(w)$ and $\overline{M}(w)$ in parallel
If $M(w)$ accepts then *accept* EndIf
If $\overline{M}(w)$ accepts then *reject* EndIf

TRUE/FALSE Questions

11) A NTM is a decider if and only if all branches of computation halt on all inputs.

TRUE

By definition.

What is wrong with the statement?

TM M rejects language L.

It is meaningless. We have not defined (nor will we) what it means for a language to be rejected by a TM (or a FA or a PDA). We have only defined what it means for a STRING to be rejected by a TM.