University of California, San Diego
Department of Computer Science and Engineering
Course: CSE 105 − Theory of Computability

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Solutions to Class Problems I

Problem 1 Write a short informal English description of each set.

- $\{ n : n = 2m \text{ for some } m \in \mathbb{N} \}$
- $\{ w : w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w \}$
- $\{ n : n \text{ is an integer and } n = n + 1 \}$

Solution

- The set of even natural numbers
- The set of palindromes over the binary alphabet
- The empty set

Problem 2 Prove that for any sets A and B, $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

Solution

Let $x \in \overline{A \cup B}$. By the definition of the complement of a set, $x \notin A \cup B$. By the definition of the union of two sets, this means that $x \notin A$ and $x \notin B$. By the definition of the complement of a set, we have $x \in \overline{A}$ and $x \in \overline{B}$. By the definition of the intersection of two sets, $x \in \overline{A} \cap \overline{B}$. (This proves that $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$.)

Let $x \in \overline{A} \cap \overline{B}$. By the definition of the intersection of two sets, $x \in \overline{A}$ and $x \in \overline{B}$. By the definition of the complement of a set, we have $x \notin A$ and $x \notin B$. By the definition of the union of two sets, this means that $x \notin A \cup B$. By the definition of the complement of a set, $x \in \overline{A \cup B}$. (This proves that $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.)

Thus every element of $\overline{A \cup B}$ is in $\overline{A} \cap \overline{B}$ and every element of $\overline{A} \cap \overline{B}$ is in $\overline{A \cup B}$. This means that the two sets are equal.

Problem 3 Find the error in the following proof that $2 = 1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

Solution

The error is in the step where each side is divided by $(a - b)$. If $a = b = 1$, then $(a - b) = 0$ and we cannot conclude from $(a + b)(a - b) = b(a - b)$ that $a + b = b$.