CSE 101 - ALGORITHMS - SUMMER 2000
Homework 1

Due Wednesday, July 12th, 8am in the class. No exception!

Problem 1: (Merge Sort)

(10 points) The following algorithm for merge sort was presented in the first lecture:

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MERGE-SORT($A, i, k$)
if $i \geq k$ then return
$j \leftarrow [\frac{i+k}{2}]$
MERGE-SORT($A, i, j$)
MERGE-SORT($A, j+1, k$)
MERGE($A, i, j, k$)
```

where MERGE(A, i, j, k) assumes that the subarrays $A[i, ..., j]$ and $A[j+1, ..., k]$ are sorted and returns all the $k-i+1$ elements in a single sorted subarray that replaces the current subarray $A[i, ..., k]$. Write and analyze pseudocode for MERGE(A, i, j, k).

Problem 2: (Function Order - Skiena, Exercise 1-7)

(10 points) Give a proof or counter-example to the following claim: for all functions f and g, either $f(n) = O(g(n))$ or $g(n) = O(f(n))$.

Problem 3: (Recurrence)

(10 points) This problem has three parts:

1. (2 points) Solve the recurrence $T(n) = T(n/2) + n$.
2. (5 points) You are a young scientist who just got a new job in a large team of 100 people (you the 101-st). A friend of yours who you believe told you that you have more honest colleagues than liars, and that that's all what he can tell you, where a *liar* is a person who can either lie or tell the truth, while an *honest* person is one who always tells the truth. Of course, you'd like to know exactly your honest colleagues and the liars, so that you decide to start an investigation, consisting of a series of questions you are going to ask your colleagues. Since you don't wish to look suspicious, you decide to ask only questions of the form "Is Mary an honest person?" and of course, to ask as few questions as possible. Can you sort out all your honest colleagues? What's the minimum number of questions you'd ask in the worst case? You can assume that your colleagues know each other well enough to say if another person is a liar or not. (Hint: Group people in pairs (X,Y) and ask X the question "Is Y honest?" and Y the question "Is X honest?". Analyze all the four possible answers. Once you find an honest person, you can easily find all the others. Challenge: can you solve this enigma asking less than 280 questions in total?)
3. (3 points) Generalize the strategy above and show that given n people such that less then half are liars, you can sort them out in honest persons and liars by asking $\Theta(n)$ questions.

Problem 4: (Sorting)

(10 points) Given a fixed $k > 0$, show how to sort any n integers in the range 1 to n^k in $O(n)$ time. (Hint: do it for $k = 2$ first; you get 8 points if you solve it for $k = 2$.)