## Control Flow Graphs

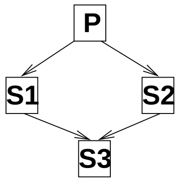
Nodes *Statements or Basic Blocks*

*(Maximal sequence of code with branching only allowed at end)*

Edges *Possible transfer of control*

**Example:**

**if P**
**then S1**
**else S2**
**S3**




**CFG**

*P predecessor of S1 and S2*
*S1, S2 sucessors of P*



## Finding Basic Blocks

Identify Headers

*The first instruction is a header*

*The target of any branch is a header*

*The instruction following any branch is a header*

*Add new nodes Entry, Exit as headers*

For each header, add successive instructions to BB until reach next header

**Ex.**

```
    a := 1
    b := 2
    if P then go to L1
    c := 3
L1: d := 4
    e := 5
```

## Finding Edges in CFG

There is a directed edge *B1* ⟶ *B2* if either:

*There is a branch from last instruction in B1 to header of B2*

*B2 immediately follows B1, and B1 does not end in an unconditional branch*

There is an edge from *Entry* to each initial BB

There is an edge from each final BB to *Exit*

There is at most one edge *B1* ⟶ *B2*



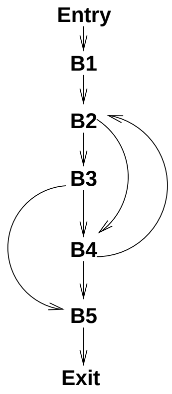
## Example

```
    a := 1
    b := 2
L1: c := a + b
    d := c - a
    if ( ) go to L2

    d := a + b
    e := e + 1
    if ( ) go to L3

L2: b := a + b
    e := c - a
    if ( ) go to L1

L3: a := b + d
    b := a - d
```

## Example

```
    a := 1     *          B1
    b := 2

L1: c := a + b   *        B2
    d := c - a
    if ( ) go to L2

    d := a + b   *        B3
    e := e + 1
    if ( ) go to L3

L2: b := a + b   *        B4
    e := c - a
    if ( ) go to L1

L3: a := b + d *          B5
    b := a - d
```

* = header






## Why are CFG's Useful?

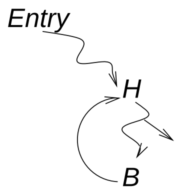
Can summarize info per BB

A pass over CFG is shorter than pass over program

Can easily find unreachable code

Makes syntactic structure (like loops) easy to find

## What are loops?




Suppose

1. All paths from *Entry* to *B* go through *H*, and *(Header H dominates B)*
2. There is an edge from *B* to *H* *(Back edge)*
3. All nodes are reachable from Entry.

**Natural Loop (wrt** $B \rightarrow H$ **)**

Nodes from which can reach *B* without going through *H*

*Can find natural loop by backward traversal from B*



## Dominators

**Def.** *A* dominates *B in CFG G iff*

*A lies on every path in G from Entry to B.*

**Facts:**

*A dominates A* **(reflexive)**

*A dominates B & B dominates C implies*
*A dominates C* **(transitive)**

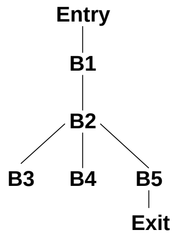
*A dominates B & B dominates A implies*
*A = B* **(anti-symmetric)**

**Def.** *A* immediately dominates *B iff A dominates B, A ≠ B, and there is no C distinct from A and B such that A dominates C and C dominates B*

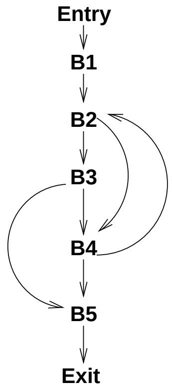
Immediate dominators form a **tree**

# Example

**Dominator Tree**




**CFG**



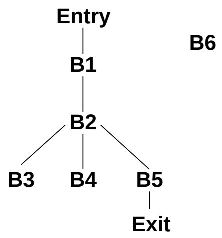

**Natural Loop (B4 → B2)**

**B2, B3, B4**

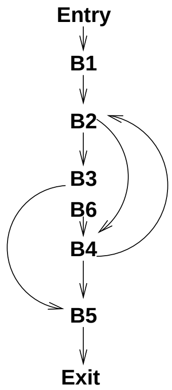


# Example

**Dominator Tree**




**CFG**

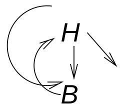



**Natural Loop (B4 → B2)**

**B2, B4**

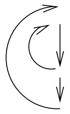
## What's wrong with natural loops?

Don't find all "loops"

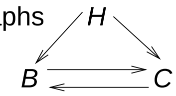



*(H doesn't dominate B)*

Don't find nesting of loops



Don't find irreducible subgraphs
*(multiple-entry SCC)*




*(No dominator relation between C and B)*

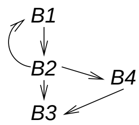


## Intervals

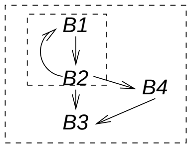
Find "Regions" in CFG
Make each region a node, and continue

*Get hierarchical nesting (possibly, control tree)*

**Ex. Cocke-Allen Intervals**

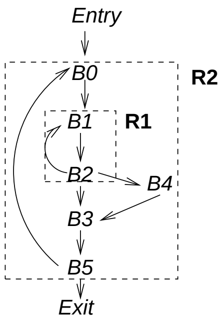



**Structural Analysis**

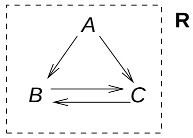


# Tarjan Intervals

**Ex.**




**Ex.**

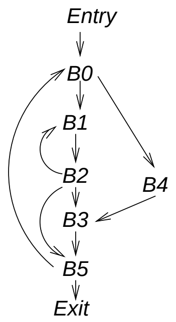



**Smallest single-entry region containing irreducible subgraph**

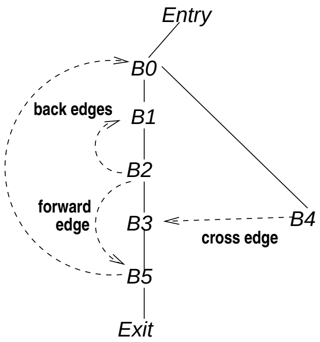


# Constructing Tarjan Intervals

**Ex. CFG**




**DFS Tree**




**Pre-order: process node before descendants**

***(Post-order: after desc.)***

**Reverse Pre-order**

# Constructing Tarjan Intervals (cont'd)

**DFS Tree**

**Back:**

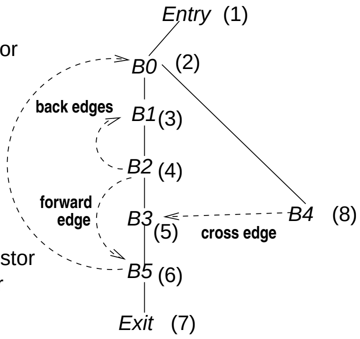
From node to an ancestor in tree

**Forward:**

From node to a descendant in tree

**Cross:**

From node to non-ancestor with smaller DF number




**Qu: How tell if ancestor or descendant?**



# Constructing Tarjan Intervals (cont'd)

**For each back edge B→ H, in reverse pre-order of headers:**

***Find Interval I(H):***

Starting from B, traverse CFG edges backwards.

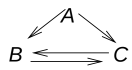
Stop when get to header H, or node with lower number than H.
*(in the latter case, the CFG is irreducible).*

The nodes traversed (including H) constitute I(H).

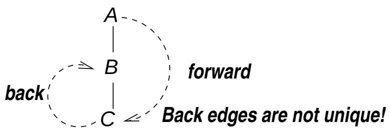
***Replace nodes in I(H) with new node $N_{I(H)}$ that points to it.***

**Result: Hierarchical CFG with Tarjan Intervals**

**Irreducible subgraph**




**DFS tree**




**Irreducible subraph is an interval!**