Department of Computer Science and Engineering
University of California at San Diego

CSE 21
Spring 97

Sample Quiz 6 Solutions

1. One fair die is tossed 1000 times.
   (a) Find the expected **sum** of all of the sixes that are thrown.

   E(number of sixes thrown) = 1000 P(6) = (1000)(1/6) = 1000/6

   E(**sum**) = 6 [E(number of sixes thrown)] = (6) (1000/6) = 1000

   (b) Find the standard deviation of the sum of the sixes that are thrown.

   σ(number of sixes thrown) = $\sqrt{n \cdot p \cdot q}$ = $\sqrt{1000 \cdot (1/6) \cdot (5/6)}$ = $50\sqrt{2}/6$

   σ(**sum**) = 6 [σ(number of sixes thrown)] = $6 \cdot (50\sqrt{2}/6)$ = $50\sqrt{2}$

2. (a) A card is drawn and replaced from an ordinary deck of cards, 5 times in succession. What is the probability that at least 4 hearts will be drawn?

   There are 13 hearts in an ordinary deck of 52 playing cards. Thus P(H) = 1/4.

   P(H $\geq$ 4) = P(H = 4) + P(H = 5) = $\binom{5}{4}(\frac{1}{4})^4(\frac{3}{4}) + \binom{5}{5}(\frac{1}{4})^5$ = 0.0156

   (b) How many fair dice must be thrown so that there is a better than even chance of obtaining a one or a two?

   In order for P(1 or 2) to be better than even, P(NOT(1 or 2)) must be worse than even ($< \frac{1}{2}$).

   P(NOT (1 or 2)) = $(\frac{2}{3})^n$ for n trials.

   n = 1, P(NOT (1 or 2)) = $(\frac{2}{3})^1 = \frac{2}{3} > \frac{1}{2}$

   n = 2, P(NOT (1 or 2)) = $(\frac{2}{3})^2 = \frac{4}{9} < \frac{1}{2}$ So, 2 dice must be thrown.

3. Suppose 1 % of blood donors are infected with HIV. Find the probability that there are at most 2 HIV contaminated blood samples amongst 500 samples taken.

   P(at most 2) = P(0) + P(1) + P(2)

   Using poisson distribution with n=500, p=0.01, then P(k) = $\frac{\lambda^k e^{-\lambda}}{k!}$ and λ = np = (500)(0.01) = 5.

   Thus P(at most 2) = $\frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!} + \frac{5^2 e^{-5}}{2!} = e^{-5} + 5e^{-5} + (\frac{25}{2})e^{-5} = (\frac{37}{2})e^{-5} \approx$ 0.125