

Sample Quiz 6 Solutions

1. One fair die is tossed 1000 times.

- (a) Find the expected **sum** of all of the sixes that are thrown.

$$E(\text{number of sixes thrown}) = 1000 P(6) = (1000)(1/6) = 1000/6$$

$$E(\text{sum}) = 6 [E(\text{number of sixes thrown})] = (6) (1000/6) = 1000$$

- (b) Find the standard deviation of the sum of the sixes that are thrown.

$$\sigma(\text{number of sixes thrown}) = \sqrt{n \cdot p \cdot q} = \sqrt{1000 \cdot (1/6) \cdot (5/6)} = 50\sqrt{2}/6$$

$$\sigma(\text{sum}) = 6 [\sigma(\text{number of sixes thrown})] = 6 \cdot (50\sqrt{2}/6) = 50\sqrt{2}$$

2. (a) A card is drawn and replaced from an ordinary deck of cards, 5 times in succession. What is the probability that at least 4 hearts will be drawn?

There are 13 hearts in an ordinary deck of 52 playing cards. Thus $P(H) = 1/4$.

$$P(H \geq 4) = P(H = 4) + P(H = 5) = \binom{5}{4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right) + \binom{5}{5} \left(\frac{1}{4}\right)^5 = 0.0156$$

- (b) How many fair dice must be thrown so that there is a better than even chance of obtaining a one or a two?

In order for $P(1 \text{ or } 2)$ to be better than even, $P(\text{NOT}(1 \text{ or } 2))$ must be worse than even ($< \frac{1}{2}$).

$$P(\text{NOT}(1 \text{ or } 2)) = \left(\frac{2}{3}\right)^n \text{ for } n \text{ trials.}$$

$$n = 1, P(\text{NOT}(1 \text{ or } 2)) = \left(\frac{2}{3}\right)^1 = \frac{2}{3} > \frac{1}{2}$$

$$n = 2, P(\text{NOT}(1 \text{ or } 2)) = \left(\frac{2}{3}\right)^2 = \frac{4}{9} < \frac{1}{2} \quad \text{So, 2 dice must be thrown.}$$

3. Suppose 1 % of blood donors are infected with HIV. Find the probability that there are at most 2 HIV contaminated blood samples amongst 500 samples taken.

$$P(\text{at most } 2) = P(0) + P(1) + P(2)$$

Using poisson distribution with $n=500$, $p=0.01$, then $P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ and $\lambda = np = (500)(0.01) = 5$.

$$\text{Thus } P(\text{at most } 2) = \frac{5^0 e^{-5}}{0!} + \frac{5^1 e^{-5}}{1!} + \frac{5^2 e^{-5}}{2!} = e^{-5} + 5e^{-5} + \left(\frac{25}{2}\right)e^{-5} = \left(\frac{37}{2}\right)e^{-5} \approx 0.125$$