Sample Quiz 5 Solutions

1. A coin is weighted so that heads is three times as likely to appear as tails. The coin is tossed 5 times. Let X denote the length of the string of heads starting in the first toss.
   Find the **(a)** distribution of X **(b)** mean of X **(c)** variance of X **(d)** standard deviation of X

   (a) f(0) = $\{\text{T????}\} = \frac{1}{4} = \frac{256}{1024}$ f(1) = $\{\text{HT???}\} = (\frac{3}{4})(\frac{1}{4}) = \frac{3}{16} = \frac{192}{1024}$

   f(2) = $\{\text{HHT??}\} = (\frac{3}{4})^2(\frac{1}{4}) = \frac{9}{64} = \frac{144}{1024}$ f(3) = $\{\text{HHHT?}\} = (\frac{3}{4})^3(\frac{1}{4}) = \frac{27}{256} = \frac{108}{1024}$

   f(4) = $\{\text{HHHHT}\} = (\frac{3}{4})^4(\frac{1}{4}) = \frac{81}{1024}$ f(5) = $\{\text{HHHHH}\} = (\frac{3}{4})^5 = \frac{243}{1024}$

X	0	1	2	3	4	5
	$\frac{256}{1024}$	$\frac{192}{1024}$	$\frac{144}{1024}$	$\frac{108}{1024}$	$\frac{81}{1024}$	$\frac{243}{1024}$

   (b) $\mu_X = 0(\frac{256}{1024}) + 1(\frac{192}{1024}) + 2(\frac{144}{1024}) + 3(\frac{108}{1024}) + 4(\frac{81}{1024}) + 5(\frac{243}{1024}) = \frac{2343}{1024} \approx 2.3$

   (c) $\sigma^2 = \frac{0(256)+1(192)+4(144)+9(108)+16(84)+25(243)}{1024} - (2.3)^2 = \frac{9111}{1024} - 5.29 \approx 3.6$

   (d) $\sigma = \sqrt{\sigma^2} = \sqrt{3.6} \approx 1.9$

2. Consider the following joint distribution of X and Y and find:
   (a) E(X) and E(Y) **(b)** Cov(X,Y) **(c)** σ_X, σ_Y, $\rho(X, Y)$.
   (d) Are X and Y are independent random variables? Defend your answer.

X \ Y	1	2	Sum
-2	0.3	0.1	0.4
1	0	0.1	0.1
3	0.1	0.2	0.3
4	0.2	0	0.2
Sum	0.6	0.4	

   (a) E(X) = μ_X = (-2)(0.4) + (1)(0.1) + (3)(0.3) + (4)(0.2) = 1.0
   E(Y) = μ_Y = (1)(0.6) + (2)(0.4) = 1.4

   (b) E(XY) = (-2)(1)(0.3) + (-2)(2)(0.1) + (1)(1)(0.0) + (1)(2)(0.1)
   + (3)(1)(0.1) + (3)(2)(0.2) + (4)(1)(0.2) + (4)(2)(0.0) = 1.5
   Cov(X,Y) = E(XY) - $\mu_X\mu_Y$ = 1.5 - 1.4 = 0.1

   (c) $\sigma_X^2 = (-2)^2(0.4) + (1)^2(0.1) + (3)^2(0.3) + (4)^2(0.2) - (\mu_X)^2 = (7.6) - (1.0)^2 = 6.6 \Rightarrow \sigma_X \approx 2.6$

   $\sigma_Y^2 = (1)^2(0.6) + (2)^2(0.4) - (\mu_Y)^2 = (2.2) - (1.4)^2 = 0.24$

   $\rho(X, Y) = \frac{\text{Cov}(X,Y)}{\sigma_X\sigma_Y} \approx \frac{0.1}{(2.6)(0.5)} \approx 0.08$

   (d) NO. for example, P(X=-2, Y=1) $\neq$ P(X=-2) P(Y=1) $\{$because 0.3 $\neq$ 0.24$\}$

3. A player tosses a pair of fair dice. Depending upon the sum of the toss, the player wins money. If the sum is 8 or higher, he wins \$8; exactly 7, he wins \$5; exactly 2, he wins \$4; other sums, he wins \$1. If the game is to be fair, how much should he pay to play the game? Round your answer to the nearest cent.

   sums from all possible rolls of two dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

   sum $\geq$ 8 for 15 dice pairs, sum = 7 for 6 pairs, sum = 2 for 1 pair, and other sums for 14 pairs.

   \$

\$8	\$5	\$4	\$1
$\frac{15}{36}$	$\frac{6}{36}$	$\frac{1}{36}$	$\frac{14}{36}$

   E(\$) = $8(\frac{15}{36}) + 5(\frac{6}{36}) + 4(\frac{1}{36}) + 1(\frac{14}{36}) = \frac{168}{36} \approx$ \$4.67