

Sample Quiz 5 Solutions

1. A coin is weighted so that heads is three times as likely to appear as tails. The coin is tossed 5 times. Let X denote the length of the string of heads starting in the first toss. Find the **(a)** distribution of X **(b)** mean of X **(c)** variance of X **(d)** standard deviation of X

$$\begin{aligned} \text{(a)} \quad f(0) &= \{T????\} = \frac{1}{4} = \frac{256}{1024} & f(1) &= \{HT???\} = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{3}{16} = \frac{192}{1024} \\ f(2) &= \{HHT??\} = \left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{9}{64} = \frac{144}{1024} & f(3) &= \{HHHT?\} = \left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) = \frac{27}{256} = \frac{108}{1024} \\ f(4) &= \{HHHHT\} = \left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) = \frac{81}{1024} & f(5) &= \{HHHHH\} = \left(\frac{3}{4}\right)^5 = \frac{243}{1024} \end{aligned}$$

X	0	1	2	3	4	5
	$\frac{256}{1024}$	$\frac{192}{1024}$	$\frac{144}{1024}$	$\frac{108}{1024}$	$\frac{81}{1024}$	$\frac{243}{1024}$

$$\text{(b)} \quad \mu_X = 0\left(\frac{256}{1024}\right) + 1\left(\frac{192}{1024}\right) + 2\left(\frac{144}{1024}\right) + 3\left(\frac{108}{1024}\right) + 4\left(\frac{81}{1024}\right) + 5\left(\frac{243}{1024}\right) = \frac{2343}{1024} \approx 2.3$$

$$\text{(c)} \quad \sigma^2 = \frac{0(256) + 1(192) + 4(144) + 9(108) + 16(81) + 25(243)}{1024} - (2.3)^2 = \frac{9111}{1024} - 5.29 \approx 3.6$$

$$\text{(d)} \quad \sigma = \sqrt{\sigma^2} = \sqrt{3.6} \approx 1.9$$

2. Consider the following joint distribution of X and Y and find:

- (a)** $E(X)$ and $E(Y)$ **(b)** $\text{Cov}(X, Y)$ **(c)** $\sigma_X, \sigma_Y, \rho(X, Y)$.
(d) Are X and Y independent random variables? Defend your answer.

$X \backslash Y$	1	2	Sum
-2	0.3	0.1	0.4
1	0	0.1	0.1
3	0.1	0.2	0.3
4	0.2	0	0.2
Sum	0.6	0.4	

$$\text{(a)} \quad E(X) = \mu_X = (-2)(0.4) + (1)(0.1) + (3)(0.3) + (4)(0.2) = 1.0$$

$$E(Y) = \mu_Y = (1)(0.6) + (2)(0.4) = 1.4$$

$$\begin{aligned} \text{(b)} \quad E(XY) &= (-2)(1)(0.3) + (-2)(2)(0.1) + (1)(1)(0.0) + (1)(2)(0.1) \\ &\quad + (3)(1)(0.1) + (3)(2)(0.2) + (4)(1)(0.2) + (4)(2)(0.0) = 1.5 \\ \text{Cov}(X, Y) &= E(XY) - \mu_X \mu_Y = 1.5 - 1.4 = 0.1 \end{aligned}$$

$$\text{(c)} \quad \sigma_X^2 = (-2)^2(0.4) + (1)^2(0.1) + (3)^2(0.3) + (4)^2(0.2) - (\mu_X)^2 = (7.6) - (1.0)^2 = 6.6 \Rightarrow \sigma_X \approx 2.6$$

$$\sigma_Y^2 = (1)^2(0.6) + (2)^2(0.4) - (\mu_Y)^2 = (2.2) - (1.4)^2 = 0.24$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} \approx \frac{0.1}{(2.6)(0.5)} \approx 0.08$$

- (d)** NO. for example, $P(X=-2, Y=1) \neq P(X=-2)P(Y=1)$ {because $0.3 \neq 0.24$ }

3. A player tosses a pair of fair dice. Depending upon the sum of the toss, the player wins money. If the sum is 8 or higher, he wins \$8; exactly 7, he wins \$5; exactly 2, he wins \$4; other sums, he wins \$1. If the game is to be fair, how much should he pay to play the game? Round your answer to the nearest cent.

sums from all possible rolls of two dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

sum ≥ 8 for 15 dice pairs, sum = 7 for 6 pairs, sum = 2 for 1 pair, and other sums for 14 pairs.

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\$8	\$5	\$4	\$1
$\frac{15}{36}$	$\frac{6}{36}$	$\frac{1}{36}$	$\frac{14}{36}$

$$E(\$) = 8\left(\frac{15}{36}\right) + 5\left(\frac{6}{36}\right) + 4\left(\frac{1}{36}\right) + 1\left(\frac{14}{36}\right) = \frac{168}{36} \approx \$4.67$$