Sample Quiz 4 Solutions

1. In a certain town, 25% of the citizens drive red cars, 45% of the citizens have received a speeding ticket, and 20% have both received a speeding ticket and drive red cars. A citizen is selected at random.

   (a) If she drives a red car, what is the probability that she has received a speeding ticket?

   $$P(S \mid R) = \frac{P(S \cap R)}{P(R)} = \frac{0.20}{0.25} = \frac{20}{25} = \frac{4}{5} = 80\%$$

   (b) What is the probability that she drives a red car if she did *not* receive a speeding ticket?

   $$P(R \mid S^c) = \frac{P(R \cap S^c)}{P(S^c)} = \frac{P(R) - P(R \cap S)}{P(S^c)} = \frac{0.25 - 0.20}{1 - 0.45} = \frac{1}{11} = 9.09\%$$

2. There are two urns with the following initial distribution:
   Urn A contains 7 red marbles and 3 white marbles
   Urn B contains 2 red marbles and 4 white marbles

   First, one marble is drawn from A and placed into B.
   Second, another marble is drawn from A and placed into B.
   Third, one marble is drawn from B.

   (a) What is the probability that all three marbles drawn were the same color?

   Let the notation A(7:2) indicate that there are 7 reds and 2 whites in urn A.

   Case 1: Three Reds
   Initially A(7:3) and B(2:4). Red is chosen from A with P(R) = $\frac{7}{10}$ and placed into B
   Now A(6:3) and B(3:4). Red is chosen from A with P(R) = $\frac{6}{9}$ and placed into B
   Now A(5:3) and B(4:4). Red is chosen from B with P(R) = $\frac{4}{8}$

   Case 2: Three Whites
   Initially A(7:3) and B(2:4). White is chosen from A with P(R) = $\frac{3}{10}$ and placed into B
   Now A(7:2) and B(2:5). White is chosen from A with P(R) = $\frac{2}{9}$ and placed into B
   Now A(7:1) and B(2:6). White is chosen from B with P(R) = $\frac{6}{8}$

   Answer: P(RRR) + P(WWW) = $(\frac{7}{10})(\frac{6}{9})(\frac{4}{8}) + (\frac{3}{10})(\frac{2}{9})(\frac{6}{8}) = (\frac{7}{30}) + (\frac{1}{20}) = (\frac{17}{60}) = 28\frac{1}{3}\%$

   (b) What is the probability of drawing at most two reds if the first marble drawn is red?

   Answer: $\frac{P(RWW) + P(RWR) + P(RRW)}{P(R??)} = [(\frac{7}{10})(\frac{3}{9})(\frac{5}{8}) + (\frac{7}{10})(\frac{3}{9})(\frac{3}{8}) + (\frac{7}{10})(\frac{6}{9})(\frac{4}{8})] \div (\frac{7}{10})$
   $= (\frac{3}{9})(\frac{4}{8}) + (\frac{3}{9})(\frac{3}{8}) + (\frac{6}{9})(\frac{4}{8}) = (\frac{12}{72}) + (\frac{9}{72}) + (\frac{24}{72}) = \frac{45}{72} = \frac{5}{8} = 62.5\%$

3. Consider a 5-day weather pattern where it is equally likely to be rainy as it is to be sunny
   Let A = event that three or more days are rainy.
   Let B = event that the first two days are sunny.
   Let C = event that all days have the same weather.

   (a) Find the probability of each event.

   $$P(A) = \frac{P(3R) + P(4R) + P(5R)}{\text{All Possible Weather}} = \frac{\binom{5}{3}+\binom{5}{4}+\binom{5}{5}}{2^5} = \frac{10+5+1}{32} = \frac{1}{2}$$

   $$P(B) = P(RR???) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

   $$P(C) = P(5R) + P(5S) = \left(\frac{1}{32}\right) + \left(\frac{1}{32}\right) = \frac{1}{16}$$

   (b) For each pair of events, decide whether they are independent. Defend your answer.

   NO: $P(A \cap B) \neq P(A)\,P(B)$
   [$A \cap B = \{SSRRR\} \Longrightarrow P(A \cap B) = \frac{1}{32}$]
   [$P(A)\,P(B) = \left(\frac{1}{2}\right)\left(\frac{1}{4} = \frac{1}{8}\right.$]

   YES: $P(A \cap C) \neq P(A)\,P(C)$
   [$A \cap C = \{RRRRR\} \Longrightarrow P(A \cap C) = \frac{1}{32}$]
   [$P(A)\,P(C) = \left(\frac{1}{2}\right)\left(\frac{1}{16} = \frac{1}{32}\right.$]

   NO: $P(B \cap C) \neq P(B)\,P(C)$
   [$B \cap C = \{SSSSS\} \Longrightarrow P(B \cap C) = \frac{1}{32}$]
   [$P(B)\,P(C) = \left(\frac{1}{4}\right)\left(\frac{1}{16} = \frac{1}{64}\right.$]