

Sample Quiz 4 Solutions

1. In a certain town, 25% of the citizens drive red cars, 45% of the citizens have received a speeding ticket, and 20% have both received a speeding ticket and drive red cars. A citizen is selected at random.

(a) If she drives a red car, what is the probability that she has received a speeding ticket?

$$P(S | R) = \frac{P(S \cap R)}{P(R)} = \frac{0.20}{0.25} = \frac{20}{25} = \frac{4}{5} = 80\%$$

(b) What is the probability that she drives a red car if she did *not* receive a speeding ticket?

$$P(R | S^c) = \frac{P(R \cap S^c)}{P(S^c)} = \frac{P(R) - P(R \cap S)}{P(S^c)} = \frac{0.25 - 0.20}{1 - 0.45} = \frac{1}{11} = 9.09\%$$

2. There are two urns with the following initial distribution:

Urn A contains 7 red marbles and 3 white marbles

Urn B contains 2 red marbles and 4 white marbles

First, one marble is drawn from A and placed into B.

Second, another marble is drawn from A and placed into B.

Third, one marble is drawn from B.

(a) What is the probability that all three marbles drawn were the same color?

Let the notation A(7:2) indicate that there are 7 reds and 2 whites in urn A.

Case 1: Three Reds

Initially A(7:3) and B(2:4). Red is chosen from A with $P(R) = \frac{7}{10}$ and placed into B

Now A(6:3) and B(3:4). Red is chosen from A with $P(R) = \frac{6}{9}$ and placed into B

Now A(5:3) and B(4:4). Red is chosen from B with $P(R) = \frac{4}{8}$

Case 2: Three Whites

Initially A(7:3) and B(2:4). White is chosen from A with $P(R) = \frac{3}{10}$ and placed into B

Now A(7:2) and B(2:5). White is chosen from A with $P(R) = \frac{2}{9}$ and placed into B

Now A(7:1) and B(2:6). White is chosen from B with $P(R) = \frac{6}{8}$

$$\text{Answer: } P(RRR) + P(WWW) = \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{4}{8}\right) + \left(\frac{3}{10}\right)\left(\frac{2}{9}\right)\left(\frac{6}{8}\right) = \left(\frac{7}{30}\right) + \left(\frac{1}{20}\right) = \left(\frac{17}{60}\right) = 28\frac{1}{3}\%$$

(b) What is the probability of drawing at most two reds if the first marble drawn is red?

$$\begin{aligned} \text{Answer: } \frac{P(RWW) + P(RWR) + P(RRW)}{P(R??)} &= \left[\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\left(\frac{5}{8}\right) + \left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\left(\frac{3}{8}\right) + \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{4}{8}\right)\right] \div \left(\frac{7}{10}\right) \\ &= \left(\frac{3}{9}\right)\left(\frac{4}{8}\right) + \left(\frac{3}{9}\right)\left(\frac{3}{8}\right) + \left(\frac{6}{9}\right)\left(\frac{4}{8}\right) = \left(\frac{12}{72}\right) + \left(\frac{9}{72}\right) + \left(\frac{24}{72}\right) = \frac{45}{72} = \frac{5}{8} = 62.5\% \end{aligned}$$

3. Consider a 5-day weather pattern where it is equally likely to be rainy as it is to be sunny

Let A = event that three or more days are rainy.

Let B = event that the first two days are sunny.

Let C = event that all days have the same weather.

- (a) Find the probability of each event.

$$P(A) = \frac{P(3R) + P(4R) + P(5R)}{\text{All Possible Weather}} = \frac{\binom{5}{3} + \binom{5}{4} + \binom{5}{5}}{2^5} = \frac{10+5+1}{32} = \frac{1}{2}$$

$$P(B) = P(RR???) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$P(C) = P(5R) + P(5S) = \left(\frac{1}{32}\right) + \left(\frac{1}{32}\right) = \frac{1}{16}$$

- (b) For each pair of events, decide whether they are independent. Defend your answer.

NO: $P(A \cap B) \neq P(A) P(B)$

$$[A \cap B = \{SSRRR\} \implies P(A \cap B) = \frac{1}{32}]$$

$$[P(A) P(B) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}]$$

YES: $P(A \cap C) \neq P(A) P(C)$

$$[A \cap C = \{RRRRR\} \implies P(A \cap C) = \frac{1}{32}]$$

$$[P(A) P(C) = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = \frac{1}{32}]$$

NO: $P(B \cap C) \neq P(B) P(C)$

$$[B \cap C = \{SSSSS\} \implies P(B \cap C) = \frac{1}{32}]$$

$$[P(B) P(C) = \left(\frac{1}{4}\right)\left(\frac{1}{16}\right) = \frac{1}{64}]$$