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Sample Quiz 3 Solutions

1. Suppose you toss five fair dice at the same time. What is the probability p that

   (a) you roll a 5-of-a-kind (e.g. 44444, called a yahtzee)?

   $$\frac{\text{\# yahtzee}}{\text{\# all possible}} = \frac{6}{6^5} = \frac{1}{6^4}$$

   (b) your roll contains 3 alike *and* 2 alike that do not match the 3 alike (e.g. 51551, called a full-house)?

   $$\frac{\text{(choose the 3 alike value) (choose the 2 alike value) (place the 2 alike)}}{\text{(\# all possible)}} = \frac{\binom{6}{1}\binom{5}{1}\binom{5}{2}}{6^4}$$

2. There are 20 men and 10 women at a Super Bowl Party. $\frac{2}{5}$ of the women and $\frac{1}{5}$ of the men are not football fans. Half of the remaining women and half of the remaining men are Patriots fans. Find the probability p that

   (a) a person chosen at random is NOT a football fan.

   It is possible from the above information to conclude the following values:
   women: 3 are Patriots fans, 4 are not football fans, leaving 3 non-Patriots football fans
   men: 8 are Patriots fans, 4 are not football fans, leaving 8 non-Patriots football fans

   So, the probability that a person chosen at random is not a football fan is
   $\frac{\text{\# non-football fans}}{\text{\# people at party}} = \frac{(4+4)}{30} = \frac{4}{15}$

   (b) a person chosen at random is a woman OR is a Patriots fan.

   Method One: Directly from the above numbers: (10 women) + (8 men Patriots fans)
   giving $p = \frac{18}{30} = \frac{3}{5}$

   Method Two: Using P(Woman $\cup$ PatsFan) = P(Woman) + P(PatsFan) - P(Woman $\cap$ PatsFan)
   giving $p = \frac{10}{30} + \frac{11}{30} - \frac{3}{30} = \frac{18}{30} = \frac{3}{5}$

   (c) when 4 people are chosen at random, exactly 2 are Patriots fans.

   There are 11 Patriots fans, leaving 19 non-Patriots Fans.

   $$\frac{\text{(choose the 2 Patriots Fans) (choose the 2 non-Patriots fans)}}{\text{(choose any 4 people at party)}} = \frac{\binom{11}{2}\binom{19}{2}}{\binom{30}{4}}$$

3. A coin of diameter $\frac{2}{3}$ -inch is tossed onto an infinite checkerboard with 1-inch squares. What is the probability p that

   (a) the coin lies completely within one square?

   We will answer this question in terms of where the center of the coin may be placed on ONE square. Since the board is infinite, the relative area that answers the question for one square represents the relative area that answers the question for the entire checkerboard.

   So, the interesting subportions of this one-inch square are those nine disjoint subsquares that are the result of dividing the square in third both lengthwise and widthwise (i.e., each subsquare measures 1/3 x 1/3 -inch, or 1/9 in^2). For ease of reference, the nine subsquares could be thought of as the components of a 3 x 3 array, with subsquare S[1,1] being the top leftmost, and S[3,3] being the bottom rightmost.

   The radius of the coin is 1/3-inch, which means that the center of the coin must be at all times 1/3-inch away from all edges in order for the coin to be entirely within one square. So the only region that works is the middle region S[2,2], which is 1/9 of the square. So, the probability p is

$$\frac{\text{(area where center of coin should land in order to be entirely within one square)}}{\text{(area where center of coin may land)}} = \frac{1/9}{1} = \frac{1}{9}$$

   (b) the coin lies only on two squares?

   (Obviously, S[2,2] is not part of this answer.) If the center of the coin lands in any of the four center edge subsquares, S[1,2] S[2,1] S[2,3] S[3,2], then the coin will lie on two squares. These are the only portions of the entire square that has the coin landing on only two squares. If the center of the coin lands in any of the corner subsquares, then the coin will lie on either three or four squares. Thus the probability p that the coin lies on only two squares is $4 \cdot (1/9) = 4/9$

   (c) the coin lies on four squares?

   (Obviously, S[2,2] S[1,2] S[2,1] S[2,3] S[3,2] are not part of this answer.) This question is similar to one of the prototype questions. The center of the coin must land in the quarter circles with 1/3-inch radius, centered at each corner of the square. Thus the probability p that the coin lies on four squares is $4 \cdot (1/4) \cdot \pi(1/3)^2 = \pi/9$

   (d) the coin lies only on three squares?

   As 1, 2, 3, and 4 are the only possible numbers of squares that the coin may land on, and we have already calculated 1, 2, and 4, then 3 is just the area of the entire square minus the areas for 1, 2, and 4. This also happens to be the remaining area from the corner subsquares that was not used in part c. Either way, the probability p that the coin lies only on three squares is

$$1 - (1/9) - (4/9) - (\pi/9) = (9 - 1 - 4 - \pi)/9 = (4 - \pi)/9$$