Department of Computer Science and Engineering CSE 21
University of California at San Diego Spring 97

Quiz 2 Solutions

1. (a) A club has 23 members. How many ways can they pick 3 people to be on a committee to plan a party?

   There are 23 ways to pick the first person, 22 ways to pick the second, and 21 ways to pick the third. Since order doesn't matter here, we need to divide by the total number of ways we can permute 3 objects, i.e. 3! Hence, the total number of ways we can pick the committee is: $\frac{23 \cdot 22 \cdot 21}{3!} = \binom{23}{3}$

   (b) How many ways can they pick a president, a vice-president, and a secretary?

   Here, order *does* matter. So there are 23 ways we can pick the president, 22 ways then that we can pick the vice-president, and 21 ways to pick the treasurer. So the total number of ways is $23 \cdot 22 \cdot 21 = \frac{23!}{20!}$

2. A *word* on a computer is a sequence of 32 binary digits (bits), i.e. the digits can only take on the values of zero and one.
   e.g. 0010 1011 0001 1000 0011 1100 0101 1000

   (a) How many distinct words are there?

   Picture the 32 bits: _ _ _ ... _
   In each bit we can either put a 0 or a 1. So there are $2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2$ (32 of these) ways that we can place 0's and 1's in the bits. Hence there are 2^{32} distinct words.

   (b) If we require exactly 10 of the bits to be set (i.e. exactly 10 of the bits have the value one), then how many distinct words are there?

   Number the slots:

   -- -- -- ··· -- --
   1 2 3 ... 31 32
   Pick ten of the slots to put 1's in. There are $\binom{32}{10}$ distinct words with exactly 10 bits set.

3. Four people play a game of cards in which each person gets 13 cards. How many possible 'deals' are there?

   This is an ordered partition problem, where each partition has 13 elements. (see theorem 2.9, p. 22) Hence, there are $\frac{52!}{13!13!13!13!} = \frac{52!}{(13!)^4} = \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$ deals.