Quiz 8 White Solutions

Problem 1 Prove that if there is a circuit in a graph that starts and ends at a vertex v and if w is another vertex in the circuit, then there is a circuit in the graph that starts and ends at w.

Solution

Suppose C is a circuit in a graph G that starts and ends at a vertex v and suppose w is another vertex in the circuit. By definition the circuit has the form:

$$ve_1v_1e_2v_2...en-1vn-1e_nv$$

where
v_i are the vertices of G and
e_i are distincts edges of G.

Then $v_i = w$ for some i, $1 \leq i \leq n-1$.

So

$$w = v_iei+1v_{i+1}...e_nve_1v_1e_2...v_i = w$$

is a circuit that starts and ends at w.

Problem 2 Draw a graph with the following specified properties, or show that it does not exits:

A graph with 6 vertices of degrees 1,3,2,0,7 and 6.

Solution

The Total degree is odd(19) so, there does not exist a graph with the specified properties.

Problem 3 Refer to your quiz for the graph

1) Draw an Euler circuit or show that it cannot exist.

   Solution Euler circuits are not unique here is one:

$$a_1a_3a_7a_1a_5a_4a_1a_7a_6a_2a_1$$

2) Draw an Hamiltonian circuit or show that it cannot exist.

   Solution There is no way to "visit" a_4 and a_5 without passing twice by a_1. Therefore there is no Hamiltonian circuit.

Problem 4 Refer to your quiz for the graph

1) Find an adjacency matrix for the following graph.

   Solution

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

2) Find the number of walks of length 2 between 2 and 3.

Solution by Theorem 11.3.2 we seek the entry 2,3 of A^2.

$$A^2 = A * A = \left(\begin{array}{ccc} 1 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 1 \end{array}\right) \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right) = \begin{pmatrix} x & x & x \\ x & x & 2 \\ x & x & x \end{pmatrix}$$

where $2 = 0 \cdot 1 + 1 \cdot 1 + 1 \cdot 1$