

Quiz 8 pink Solutions

Problem 1 Prove that if there is a circuit in a graph that starts and ends at a vertex v and if w is another vertex in the circuit, then there is a circuit in the graph that starts and ends at w .

Solution

Suppose C is a circuit in a graph G that starts and ends at a vertex v and suppose w is another vertex in the circuit. By definition the circuit has the form:

$$ve_1v_1e_2v_2...e_nv_n-1v_n-1e_nv$$

where

v_i are the vertices of G and

e_i are distinct edges of G .

Then $v_i = w$ for some i , $1 \leq i \leq n-1$.

So

$$w = v_i e_i + 1 v_{i+1} \dots e_n v e_1 v_1 e_2 \dots v_i = w$$

is a circuit that starts and ends at w .

Problem 2 Draw a graph with the following specified properties, or show that it does not exist:

A graph with 6 vertices of degrees 1, 3, 2, 0, 7 and 6.

Solution

The Total degree is odd(19) so, there does not exist a graph with the specified properties.

Problem 3 Refer to your quiz for the graph

- 1) Draw an Euler circuit or show that it cannot exist.

Solution By the contrapositive of Theorem 11.2.2 if some vertex of a graph have odd degree, then the graph does not have an Euler circuit.

Since a_8 is odd. No Euler circuit exist.

- 2) Draw an Hamiltonian circuit or show that it cannot exist.

Solution Hamiltonian circuits are not unique here is one:

$$a_1 a_3 a_6 a_7 a_9 a_8 a_5 a_4 a_2 a_1$$

Problem 4 Refer to your quiz for the graph

- 1) Find an adjacency matrix for the following graph.

Solution

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

- 2) Find the number of walks of length 2 between 2 and 3.

Solution by Theorem 11.3.2 we seek the entry 2,3 of A^2 .

$$A^2 = A * A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} x & x & x & x \\ x & x & 4 & x \\ x & x & x & x \\ x & x & x & x \end{pmatrix}$$

where $4 = 0 \cdot 0 + 2 \cdot 2 + 2 \cdot 0 + 0 \cdot 0$