Quiz 6 Solutions

Problem 1 A weighted coin with$p(H) = 2/3$ is tossed 20 times. Determine the probability p that the numbers of heads occuring is between 12 and 15 by

i) Using the binomial distribution
Solution

$$b(12; 20, 2/3) = \binom{20}{12}\left(\frac{2}{3}\right)^{12}\left(\frac{1}{3}\right)^{8}$$

$$b(13; 20, 2/3) = \binom{20}{13}\left(\frac{2}{3}\right)^{12}\left(\frac{1}{3}\right)^{8}$$

$$b(14; 20, 2/3) = \binom{20}{14}\left(\frac{2}{3}\right)^{12}\left(\frac{1}{3}\right)^{8}$$

$$b(15; 20, 2/3) = \binom{20}{15}\left(\frac{2}{3}\right)^{12}\left(\frac{1}{3}\right)^{8}$$

$$p = b(12; 20, 2/3) + b(13; 20, 2/3) + b(14; 20, 2/3) + b(15; 20, 2/3)$$

ii) Using the normal approximation to the binomial distribution
Solution

$$\mu = np = 20 \cdot \left(\frac{2}{3}\right) = 13.3$$

$$\tau = \sqrt{npq} = \sqrt{20\frac{2}{3}\frac{1}{3}} = 2.1$$

$$\begin{aligned} p &\approx p(11.5 \le X \le 15.5) \\ &= p(\frac{11.5 - 13.3}{2.1} \le X^* \le \frac{15.5 - 13.3}{2.1}) \\ &= p(-0.86 \le X^* \le 1.05) \\ &= p(-0.86 \le X^* \le 0) + p(0 \le X^* \le 1.05) \\ &= p(0 \le X^* \le 0.86) + p(0 \le X^* \le 1.05) \\ &= 0.3051 + 0.3531 = 0.6582 \end{aligned}$$

i) What does this problem imply about the normal approximation.
Solution

Since p and q are both away from zero, and n is large enough (in comparison) the normal distribution is a good approximation of the binomial distribution.

Problem 2 170 students take an exam. Suppose the scores are normally distributed with mean 62 and standard deviation 4 points. If 90 is an A, 80 is a B ... and 49 and below is a failling F, determine how many people get A's. Also determine how many people fail.

Solution

$$\begin{aligned} p(X \geq 90) &= P(X^* \geq \frac{90-62}{4}) \\ &= P(X^* \geq 7) = 0 \end{aligned}$$

No students gets an A. Similarly

$$\begin{aligned} p(X \leq 49) &= P(X^* \leq \frac{49-62}{4}) \\ &= P(X^* \leq -3.25) = 0.5 - p(0 \leq X^* \leq 3.25) \\ &= 0.0006 \end{aligned}$$

Thus, the number of people who fail is $170 \cdot 0.0006 = 0.1$. No students gets a failling grade.

Problem 3 Suppose that there are 250 misprints distributed randomly throughout a book that is 225 pages. Find the probability that:

1) a given page has no misprint.

Solution

Your text propose in that case that a poisson approximation be used with $n = 500$, and $p = 1/225$ so

$\lambda = \frac{500}{225} = 1.1$

$$p(0; 1.1) = \frac{(1.1)^0 e^{-1.1}}{0!} = e^{-1.1} \approx 0.333$$

2) a given page has 2 misprints.

Solution

$$p(2; 1.1) = \frac{(1.1)^2 e^{-1.1}}{2!} \approx 0.201$$