Quiz 4 Solutions

Problem 1 Let A, B and C br three independent events with $P(A) = 1/2$, $P(C) = 1/4$ and $P(A \cap B) = 1/6$. Find:

1) $P(b)$.

Solution Since A and B are independent

$$P(A \cap B) = P(A)P(B)$$

so

$$\begin{aligned} P(B) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{\frac{1}{6}}{\frac{1}{2}} \\ &= \frac{1}{3} \end{aligned}$$

2) $P(B|A)$.

Solution

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = \frac{1}{3}$$

2) $P(A \cap B \cap C)$.

Solution by independence we have

$$P(A \cap B \cap C) = P(A)P(B)P(C) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{24}$$

Problem 2 A red die and a blue die are thrown. Find the probability p that the sum is 8 or grater if

1) a 4 appears on the blue die.

Solution so the red die can be $S = \{1, 2, 3, 4, 5, 6\}$ and $A = \{4, 5, 6\}$

$$p = \frac{1}{2}$$

2) A 4 appears on one of the two die.

Solution there are 11 possible combination (4 and 4 is only once) 41,42,43,44,45,46,14,24,34,54,64 and only 5 of them are greater or equal to 8. 44,45,54,46,64 so

$$p = \frac{5}{11}$$

Problem 3 A box contains 7 lightbulbs of which 3 are defective. The bulbs are tried 1 at a time.

1) What is the probability that the first 3 bulbs tried are all defectives.

Solution We have $BBBGGGG$ where B = defective and G = good. and we consider BBB

$$\frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{35}$$

2) What is the probability that it takes 4 tests to find all the defective bulbs.

Solution This time we consider $BBBG$ or $BGBB$ or $GBBB$ or $GGBG$ or $GGGG$ because in that case we know that the remaining bads are the one we did not pick.

$$p = 4\left(\frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{1}{4}\right) + \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{35} + \frac{1}{35} = \frac{2}{35}$$

Problem 4 A coin is weighted such that $P(H) = 2/3$. Given that a head appeared on the first toss, what is the probability that two or more heads and then a tail appears on the next 3 tosses.

Solution Let $A = \{HHT\}$ and $B = \{H\}$ We are looking for $P(A|B)$ but since the 2 events are independents we get:

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(A) \cdot P(B)}{P(B)} = P(A) \\ &= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27} \end{aligned}$$