# Quiz 3 Solutions

**Problem 1** Let $P$ be a probability function on the set $S = a, b, c, d$. Find the probability of $P(a)$ if:

1) $P(b) = 1/4, P(c) = 1/8, P(d) = 1/6$.

**Solution** By definition a probability function must =1. So we must have:

$$\begin{aligned} P(a) + P(b) + P(c) + P(d) &= 1 \\ P(a) &= 1 - (P(b) + P(c) + P(d)) \\ P(a) &= 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{6}\right) \\ P(a) &= 1 - \left(\frac{13}{24}\right) \\ P(a) &= \frac{11}{24} \end{aligned}$$

2) $P(b) = 1/2, P(c) = 1/4, P(c) = 2P(d)$.

**Solution** Similarly

$$\begin{aligned} P(a) + P(b) + P(c) + P(d) &= 1 \\ P(a) &= 1 - (P(b) + P(c) + P(d)) \\ P(a) &= 1 - (P(b) + P(c) + P(c)/2) \\ P(a) &= 1 - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \\ P(a) &= 1 - \left(\frac{7}{8}\right) \\ P(a) &= \frac{1}{8} \end{aligned}$$

**Problem 2** A man is dealt 3 cards from a deck of 52. Find the probability that.:

1) 2 are spades and one is diamond.

**Solution** We first need to look for our sample space which is $\binom{52}{3}$. Then there are 13 spades in a deck and we want to choose 2 from them. There are 13 diamonds from which we want to choose 1 of of them. So

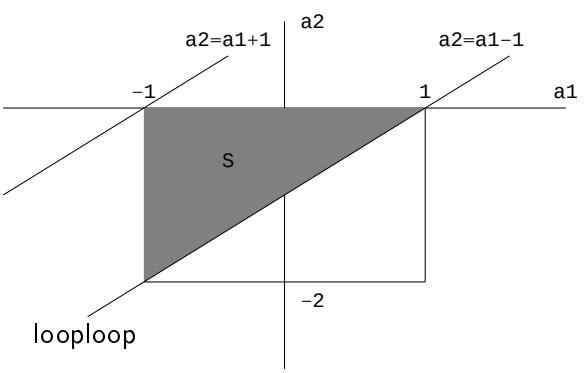
$$P(\text{2 spade and 1 diamond}) = \frac{\binom{13}{2}\binom{13}{1}}{\binom{52}{3}}$$

2) All three are diamond

**Solution**

$$P(3 \text{ diamond}) = \frac{\binom{13}{3}}{\binom{52}{3}}$$

**Problem 3** The two points $a1$ and $a2$ are selected at random from the real line such that $-1 \leq a1 \leq 1$and $-2 \leq a2 \leq 0$. Find the probability that the distance between $a1$ and $a2$ is less than 1.

**Solution** Recall that the distance between $a1$ and $a2$ is given by $d = |a2 - a1|$.In other words, $-1 \leq a2 - a1 \leq 1$ or $a2 \leq a1 + 1$ and $a2 \geq a1 - 1$. So all points inside the gray area (in the picture below) have a distance less than 1.

$$\begin{aligned} P(d \leq 1) &= \frac{Area(S)}{\text{Total Area}} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$