

Quiz 3 Solutions

Problem 1 Let P be a probability function on the set $S = a, b, c, d$. Find the probability of $P(a)$ if:

1) $P(b) = 1/4, P(c) = 1/8, P(d) = 1/6$.

Solution By definition a probability function must $= 1$. So we must have:

$$\begin{aligned}P(a) + P(b) + P(c) + P(d) &= 1 \\P(a) &= 1 - (P(b) + P(c) + P(d)) \\P(a) &= 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{6}\right) \\P(a) &= 1 - \left(\frac{13}{24}\right) \\P(a) &= \frac{11}{24}\end{aligned}$$

2) $P(b) = 1/2, P(c) = 1/4, P(c) = 2P(d)$.

Solution Similarly

$$\begin{aligned}P(a) + P(b) + P(c) + P(d) &= 1 \\P(a) &= 1 - (P(b) + P(c) + P(d)) \\P(a) &= 1 - (P(b) + P(c) + P(c)/2) \\P(a) &= 1 - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \\P(a) &= 1 - \left(\frac{7}{8}\right) \\P(a) &= \frac{1}{8}\end{aligned}$$

Problem 2 A man is dealt 3 cards from a deck of 52. Find the probability that.:

1) 2 are spades and one is diamond.

Solution We first need to look for our sample space which is $\binom{52}{3}$. Then there are 13 spades in a deck and we want to choose 2 from them. There are 13 diamonds from which we want to choose 1 of of them. So

$$P(2 \text{ spade and } 1 \text{ diamond}) = \frac{\binom{13}{2}\binom{13}{1}}{\binom{52}{3}}$$

2) All three are diamond

Solution

$$P(3 \text{ diamond}) = \frac{\binom{13}{3}}{\binom{52}{3}}$$

Problem 3 The two points a_1 and a_2 are selected at random from the real line such that $-1 \leq a_1 \leq 1$ and $-2 \leq a_2 \leq 0$. Find the probability that the distance between a_1 and a_2 is less than 1.

Solution Recall that the distance between a_1 and a_2 is given by $d = |a_2 - a_1|$. In other words, $-1 \leq a_2 - a_1 \leq 1$ or $a_2 \leq a_1 + 1$ and $a_2 \geq a_1 - 1$. So all points inside the gray area (in the picture below) have a distance less than 1.

$$\begin{aligned} P(d \leq 1) &= \frac{\text{Area}(S)}{\text{Total Area}} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

