# Quiz 2 Solutions

**Problem 1** Let $A = \{1, 3\}, B = \{C, D, E\}, C = \{2, 4, 6\}$. Use a tree diagram to find the product set $B \times A \times C$.

**Solution** The product are the following triplet: (sorry I have difficulty to make a tree image).

$$(C, 1, 2)$$
$$(C, 1, 4)$$
$$(C, 1, 6)$$
$$(C, 3, 2)$$
$$(C, 3, 4)$$
$$(C, 3, 6)$$
$$(D, 1, 2)$$
$$(D, 1, 4)$$
$$(D, 1, 6)$$
$$(D, 3, 2)$$
$$(D, 3, 4)$$
$$(D, 3, 6)$$
$$(E, 1, 2)$$
$$(E, 1, 4)$$
$$(E, 1, 6)$$
$$(E, 3, 2)$$
$$(E, 3, 4)$$
$$(E, 3, 6)$$

**Problem 2** A man is dealt a poker hand (5 cards) from a deck of 52.

1) How many different combinations can he get.

   **Solution** this is the number of possible combinations.

$$\binom{52}{5}$$

2) How many different combinations can he get if there are four cards missing.

**Solution** $52 - 4 = 48$.

$$\binom{48}{5}$$

3) How many different combinations can he get if his hand contains two aces

**Solution** A typical hand would be AAXXX where A is an Ace and X can be any card but an ace. You first should notice AAXXX is the same as AXAXX or AXXXA. So there are $\binom{4}{2}$ ways of getting to aces. Now we need to pick three others card NOT an ace and not of the same kind. i.e. The problem ask for only one pair choosing directly for the remaining 48 cards would count the possiblity of 2 kings...or even 3 aces...

So consider a color of which we remove the ace. There are $\binom{12}{3}$ ways of picking 3 cards that are of different kind. And since for each kind (3) there four different color the answer is:

$$\binom{4}{2} \cdot \binom{12}{3} \cdot 4^3$$

**Problem 3**

1) Find the number of ways in which 5 boys and 5 girls can be sited alternatively around a round table.

**Solution 1** Around a round table $BGBGBGBGBG$ is undistinguisble from $GBGBGBGBGB$. So consider the first case: there 5 ways of choosing the boy in the first position but he will have a girl on each side so there are only 4 ways of chosing the girl on his right ( or on his left). So there are $5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$ ways to arrange them or $5! \cdot 4!$.

**Solution 2** In problem 2.42 we saw that if they were arranged in a row the number of ways is: $2 \cdot 5! \cdot 5!$. If we sit them around the round table, for every arrangement there are exactly 10 equivalent ones. (i.e:$BGBGBGBGBG$ is undistinguisable from $GBGBGBGBGB$ and $B_1G_1B_2G_2B_3G_3B_4G_4B_5G_5$ from say $B_1G_1B_5G_5B_3G_3B_4G_4B_2G_2$. Sothe answer is

$$\frac{5! \cdot 5! \cdot 2}{2 \cdot 5} = 5! \cdot 4!$$

2) Find the number of ways if they sit alternatively and if one boy and one girl are to sit in adjacent seats.

**Solution** Let $BGB_1G_1BGBGBG$ be a typical sitting. As previously (because of the round table) this is undistinguisable from say $B_1GBGBGBGBG_1$. So consider for convinience $B_1G_1BGBGBGBG$. This setting is, however, distinguisable from $G_1B_1GBGBGBGB$. So there are $2 \cdot 4! \cdot 4!$ ways.

3) Find the number of ways if they sit alternatively and if one boy and one girl must not sit in adjacent seats.

**Solution** In 1) we considered all possible alternate sitting. In 2) we counted all the ones where $B_1$ and $B_2$ must be together. So $(5! \cdot 4!) - (2 \cdot 4! \cdot 4!)$ is the number of ways where $B_1$ and $B_2$ are not sitted together.