Quiz 1 Solutions

Problem 1 Let $A = B \cap C$. Prove that $A \times A = (B \times C) \cap (C \times B)$.

Solution

$$\begin{aligned} A \times A &= \{(x,y) | x \in A, y \in A\} \\ &= \{(x,y) | x \in B \quad and \quad x \in C, y \in B \quad and \quad y \in C\} \quad since \quad A = B \cap C \\ &= \{(x,y) | x \in B \quad and \quad y \in C \quad and \quad y \in B \quad and \quad x \in C\} \\ &= \{(x,y) | x \in B \quad and \quad y \in C\} \cap \{(x,y) | y \in B \quad and \quad x \in C\} \\ &= (B \times C) \cap (C \times B) \end{aligned}$$

Problem 2 Consider the following set: $A = \{c, \{t,i\}, \{d,e,f\}\}$

1) Find the power set $\wp(A)$

   Solution

   $\wp(a) = \{\emptyset, \{c\}, \{\{t,i\}\}, \{\{d,e,f\}\}, \{c, \{t,i\}\}, \{c, \{d,e,f\}\}, \{\{t,i\}, \{d,e,f\}, \}, \{c, \{t,i\}, \{d,e,f\}\}\}$

2) List 3 of the partitions of A

   Solution Note that there are only 3.

$$P_1 = [\{c\}, \{\{t,i\}\}, \{\{d,e,f\}\}]$$
$$P_2 = [\{c, \{t,i\}\}; \{\{d,e,f\}\}]$$
$$P_3 = [\{c\}; \{\{t,i\}, \{d,e,f\}\}]$$

Problem 3 Let $\mathcal{A}$ and $\mathcal{B}$ be algebras of subset of U. Is $\mathcal{A} \cup \mathcal{B}$ also an algebra of the subset of U ? Explain.

Solution Yes.

Although a formal proof was not requested you needed to say:

Let A and B be any arbitrary set such that $A \in \mathcal{A}$ and $B \in \mathcal{B}$.

1) we need to verify that the complement of $A \cup B \in \mathcal{A} \cup \mathcal{B}$. by De Morgan's law we get $(A \cup B)^c = A^c \cap B^c$. A^c is in $\mathcal{A}$ since $\mathcal{A}$ is an algebra of the subset of U. similarly for B^c. Thus the intersection is in $\mathcal{A}$ and in $\mathcal{B}$ and therefore $A \cup B \in \mathcal{A} \cup \mathcal{B}$.

2) Here we need to explain why $\mathcal{A} \cup \mathcal{B}$ is closed under union.

The only thing to note here is that since U is in $\mathcal{A}$ and $\mathcal{B}$ certainly $A \cup B \subset U$. Thus $A \cup B$ is $\mathcal{A} \cup \mathcal{B}$.