

Quiz 1 Solutions

Problem 1 Let $A = B \cap C$. Prove that $A \times A = (B \times C) \cap (C \times B)$.

Solution

$$\begin{aligned} A \times A &= \{(x, y) | x \in A, y \in A\} \\ &= \{(x, y) | x \in B \text{ and } x \in C, y \in B \text{ and } y \in C\} \text{ since } A = B \cap C \\ &= \{(x, y) | x \in B \text{ and } y \in C \text{ and } y \in B \text{ and } x \in C\} \\ &= \{(x, y) | x \in B \text{ and } y \in C\} \cap \{(x, y) | y \in B \text{ and } x \in C\} \\ &= (B \times C) \cap (C \times B) \end{aligned}$$

Problem 2 Consider the following set: $A = \{c, \{t, i\}, \{d, e, f\}\}$

- 1) Find the power set $\wp(A)$

Solution

$$\wp(A) = \{\emptyset, \{c\}, \{\{t, i\}\}, \{\{d, e, f\}\}, \{c, \{t, i\}\}, \{c, \{d, e, f\}\}, \{\{t, i\}, \{d, e, f\}\}, \{c, \{t, i\}, \{d, e, f\}\}\}$$

- 2) List 3 of the partitions of A

Solution Note that there are only 3.

$$P_1 = [\{c\}, \{\{t, i\}\}, \{\{d, e, f\}\}]$$

$$P_2 = [\{c, \{t, i\}\}, \{\{d, e, f\}\}]$$

$$P_3 = [\{c\}, \{\{t, i\}, \{d, e, f\}\}]$$

Problem 3 Let $\mathcal{A}$ and $\mathcal{B}$ be algebras of subset of U . Is $\mathcal{A} \cup \mathcal{B}$ also an algebra of the subset of U ? Explain.

Solution Yes.

Although a formal proof was not requested you needed to say:

Let A and B be any arbitrary set such that $A \in \mathcal{A}$ and $B \in \mathcal{B}$.

- 1) we need to verify that the complement of $A \cup B \in \mathcal{A} \cup \mathcal{B}$. by De Morgan's law we get

$(A \cup B)^c = A^c \cap B^c$. A^c is in $\mathcal{A}$ since $\mathcal{A}$ is an algebra of the subset of U . similarly for B^c . Thus the intersection is in $\mathcal{A}$ and in $\mathcal{B}$ and therefore $A \cup B \in \mathcal{A} \cup \mathcal{B}$.

2) Here we need to explain why $\mathcal{A} \cup \mathcal{B}$ is closed under union.

The only thing to note here is that since U is in $\mathcal{A}$ and $\mathcal{B}$ certainly $A \cup B \subset U$. Thus $A \cup B$ is $\mathcal{A} \cup \mathcal{B}$.