CSE 20, Discrete Mathematics, Spring, 1997
Relations, Equivalence Relations and Partial Orders
(Chapter 10)
Homework 4, Due. May 29.

Short answers (2 points each):

Section 10.2, pp554-555 10, 39

Section 10.3, p. 571-572 14b.,16,

Section 10.5, pp600-601 16b., 20, 45(any topological sort will do)

a. Find an integer x so that $x23 \equiv_3 21$, i.e., $32|23x-1$. b. In New Hampshire, which has no sales tax, pencils cost 32 cents each and erasers cost 23 cents each. I spent a total of \$5.08 on pencils and erasers. How many pencils did I buy?

Compute the remainder when $1,000,000,000,000,000,000,000,000,000,000 = 10^{30}$ is divided by 13.

Full Proofs (5pts. each):

Prefix partial order Let $S = \{0,1\}^*$ be the set of all binary words . Let xPy be the prefix relationship, i.e., xPy if and only if there is an $w \in S$ with $y = x \circ w$, where $\circ$ is the concatenation of two strings. Prove that P is a partial order. You may use the following without proof: $x = x \circ \lambda$, where λ represents the empty word (the only word of length 0); $l(x \circ a) = l(x) + l(a)$, where $l(x)$ is the (non-negative integer) length of the word, and $(x \circ a) \circ b = x \circ (a \circ b)$. (See page 239 for some definitions.)

Section 10.2. pp. 554-555 20, 35

Section 10.3, pp. 571-572 31, 37

Let R be the following relation on $Z \times Z$ defined by $(a,b)R(c,d)$ if and only if $a \leq c$ and $b \leq d$. Prove that R is a partial order. Does R have an infinite chain? An infinite anti-chain?

Let R be the relation on $Z \times Z$ defined by $(a,b)R(c,d)$ if and only if $a+b = c+d$. Prove that R is an equivalence relation. What are R's equivalence classes?

Let p be a prime and let $0 \leq a < p$. Prove that there are at most two numbers $x, 0 \leq x < p$ so that $x^2 mod p = a$.

Extra Credit problem (2 points extra credit .) 1. There are 26 people in a line to get into a theater. Prove that there are 6 people that are either in increasing or decreasing order of height from front to back.