CSE 20, Discrete Mathematics, Spring 1997
Homework 2, Due Monday, April 28

Short problems:(2pts. each, or per part.)

Section 2.1, pp 87-89 5, 15d., 25, 36

Section 2.2 pp.97-99 11, 12, 17

Section 5.1, pp. 242-243 10a.,e, 12c.,d., 14b.,17b.

Mathematical Definitions (2 points each part) An integer x divides an integer y if there is an integer z so that $xz = y$. (See section 3.3.) If we use the notation $x|y$ for x divides y, we can write this symbolically as $x|y \iff \exists z \in \mathbf{Z} xz = y$. (This defines the predicate symbol —: this definition is true for all x and all y, but we do not usually put in the universal quantifiers in a definition.) Use this notation to express the following statements in terms of predicate logic. Then replace the symbol | with its definition and simplify the underlying logical forms. You can assume the domain of all variables is the set of integers, $\mathbf{Z}$. An integer x is positive if and only if $x > 0$.

1. x is not a factor of y
2. For two integers, x and y, x is *relatively prime to* y if and only if the only positive integer that divides both x and y is 1. Express "x is relatively prime to y"
3. Express the statement: " all integers dividing x also divide y."

Full proofs, (5 pts. each.)

1. Epp, Section 2.3, 30

program correctness Use the predicates $h(x, t)$ to mean "the program halts on input x after t time steps" and let f be a function from inputs to outputs.

Assume that, for all inputs x, there is some t so that the program halts on x after t time steps, i.e. the program eventually halts on all inputs. Also assume that for all inputs x and times t, if the program halts after t time steps, then it outputs $f(x)$.

Formalize the above assumptions and use them to prove that, on any input x, the program outputs $f(x)$.

(Hint: do not get confused between the VALUE f(x), a number, and the PREDICATE "The program outputs f(x) on input x")

divisibility Prove that, for any integers x and y, if all numbers that divide x also divide y, then x divides y.

Section 3.5, p. 153 13a.,b. (5 pts. total.)

Section 5.2, 13

Section 5.2, 19