

CSE 200: Spring 2026
Homework 3
due May 18, 2026

Please start your response to each question on a new page.

Question 1 (PSPACE does not have fixed polynomial size circuits). *Recall that*

- PSPACE is the class of languages computable in polynomial space
- P/poly is the class of languages computable by polynomial size circuits

We believe that PSPACE is not a subset of P/poly, but this is open. In this question you will prove a weaker statement: PSPACE is not a subset of SIZE(n^k) for any fixed k .

Fix $k \geq 1$. Your goal is to construct a language L_k that satisfies two properties:

- (a) L_k can be decided in PSPACE. In fact, it is decided in $\text{SPACE}(n^{O(k)})$.
- (b) There exists n_0 , such that for all $n \geq n_0$, the language L_k cannot be computed by boolean circuits of size n^k .

Steps:

- (1) Fix an input length n . Let F_n be the class of functions $f : \{0, 1\}^n \rightarrow \{0, 1\}$ which can be computed by a circuit of size n^k . Prove that $|F_n| \leq 2^m$ for $m = O(n^{k+1})$.
- (2) Let $t \geq 1$, and fix distinct inputs $x_1, \dots, x_t \in \{0, 1\}^n$. Prove that there exist values $b_1, \dots, b_t \in \{0, 1\}$ such that the number of functions $f \in F_n$ that satisfy $f(x_i) = b_i$ for all i is at most 2^{m-t} .
- (3) Argue that for $t = m + 1$, there are inputs $x_1, \dots, x_t$ and values $b_1, \dots, b_t$ such that any function f which satisfies $f(x_i) = b_i$ must be outside F_n .
- (4) Prove that given an input length n , you can find such inputs and outputs in space $\text{poly}(m)$.
- (5) Complete the proof - describe L_k and prove its properties.
- (6) Show that the language L_k is in the polynomial hierarchy. What is the lowest level you can achieve?