Intensity Transformations

Image Processing

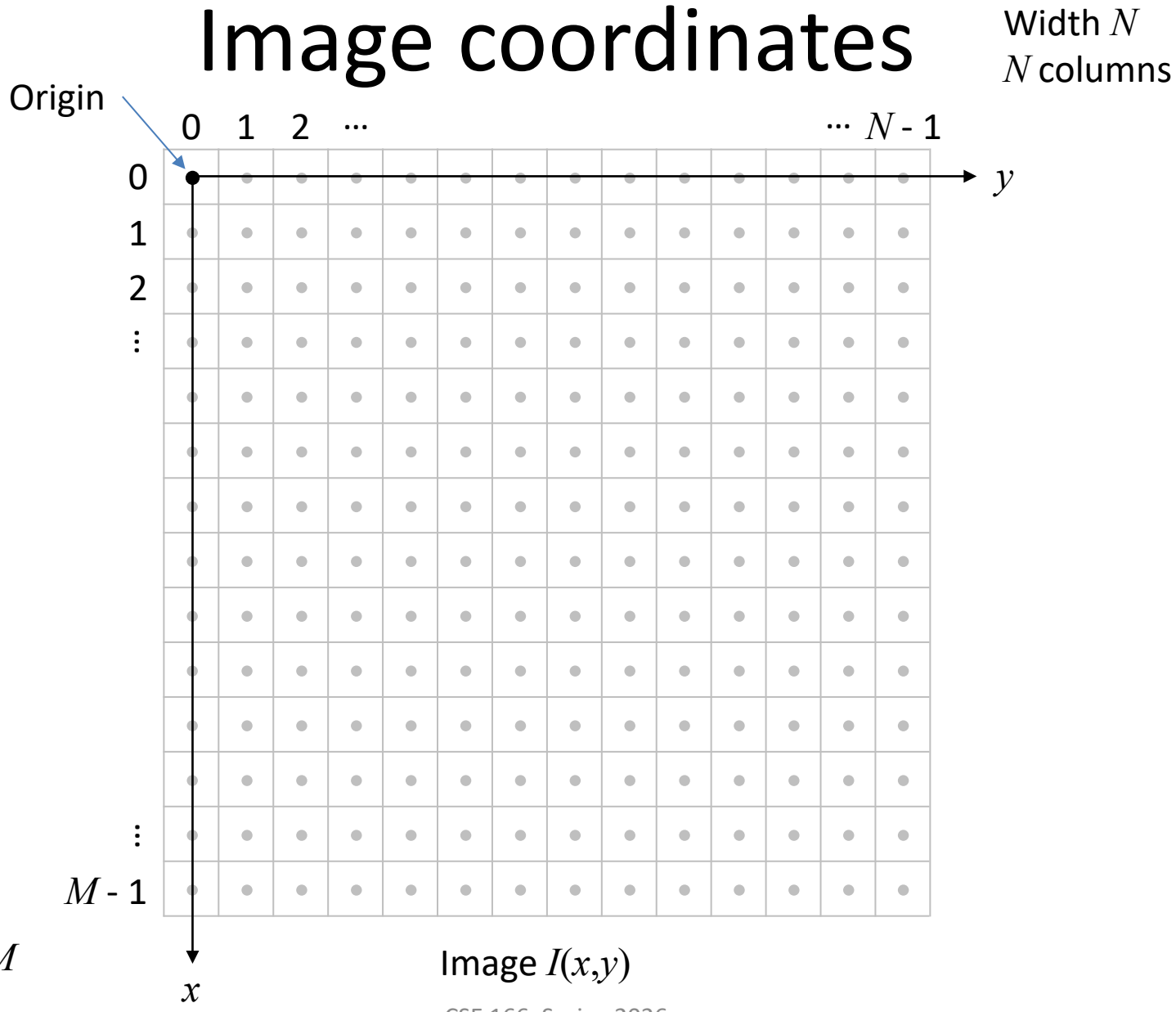
CSE 166

Lecture 3

Announcements

- Assignment 1 is due Apr 8, 11:59 PM
- Assignment 2 will be released Apr 8
  - Due Apr 15, 11:59 PM
- Reading
  - Chapter 3: Intensity Transformations and Spatial Filtering
    - Sections 3.1, 3.2, and 3.3

Image coordinates



Intensity transformations

$$g(x, y) = T(f(x, y))$$

where

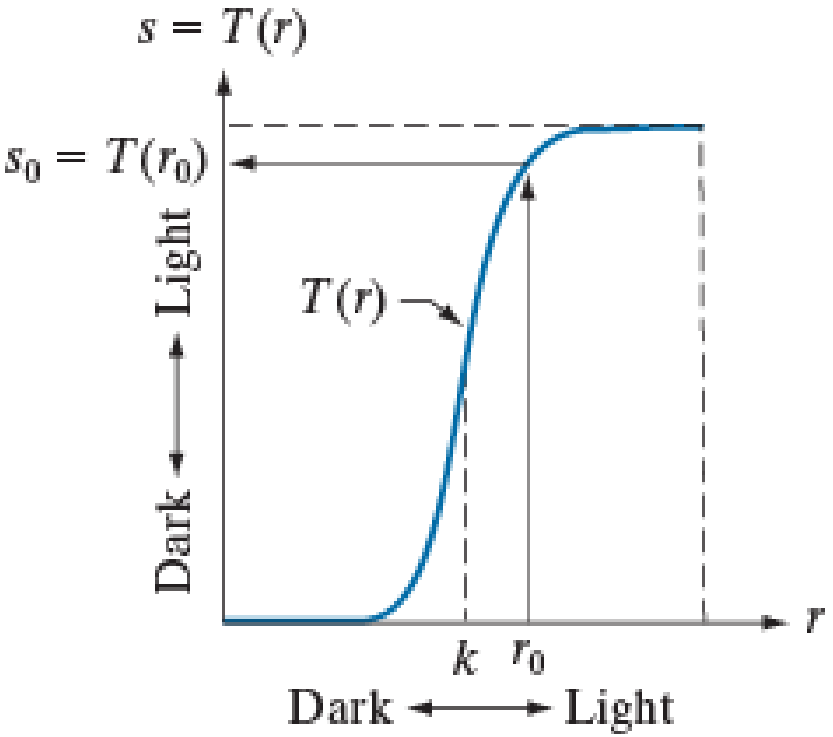
$f(x, y)$ is input image
$g(x, y)$ is output image
$T(\cdot)$ is intensity transformation

$$g(x, y) = T(f(x, y))$$
$$s = T(r)$$

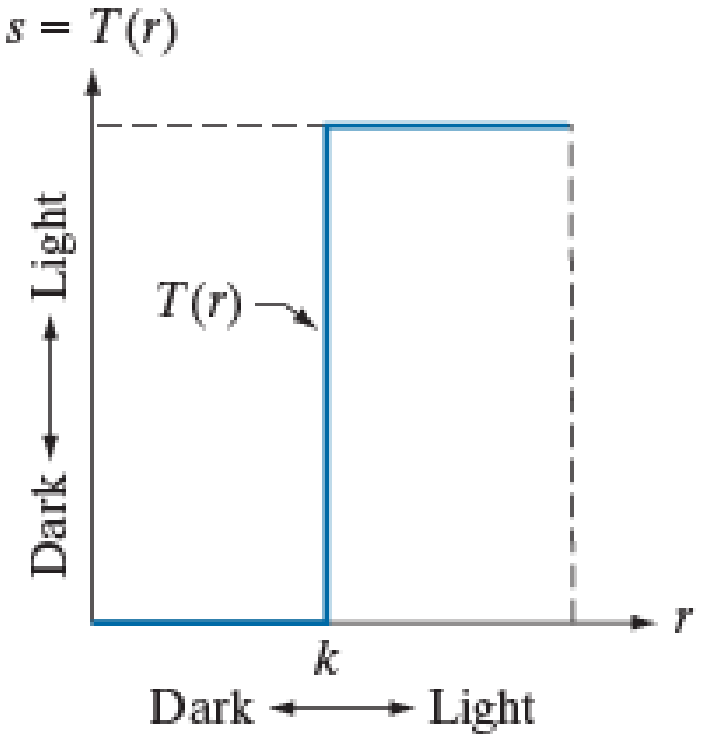
where

$r = f(x, y)$ is intensity or gray level value of input image at (x, y)
$s = g(x, y)$ is intensity or gray level value of output image at (x, y)

Intensity transformations



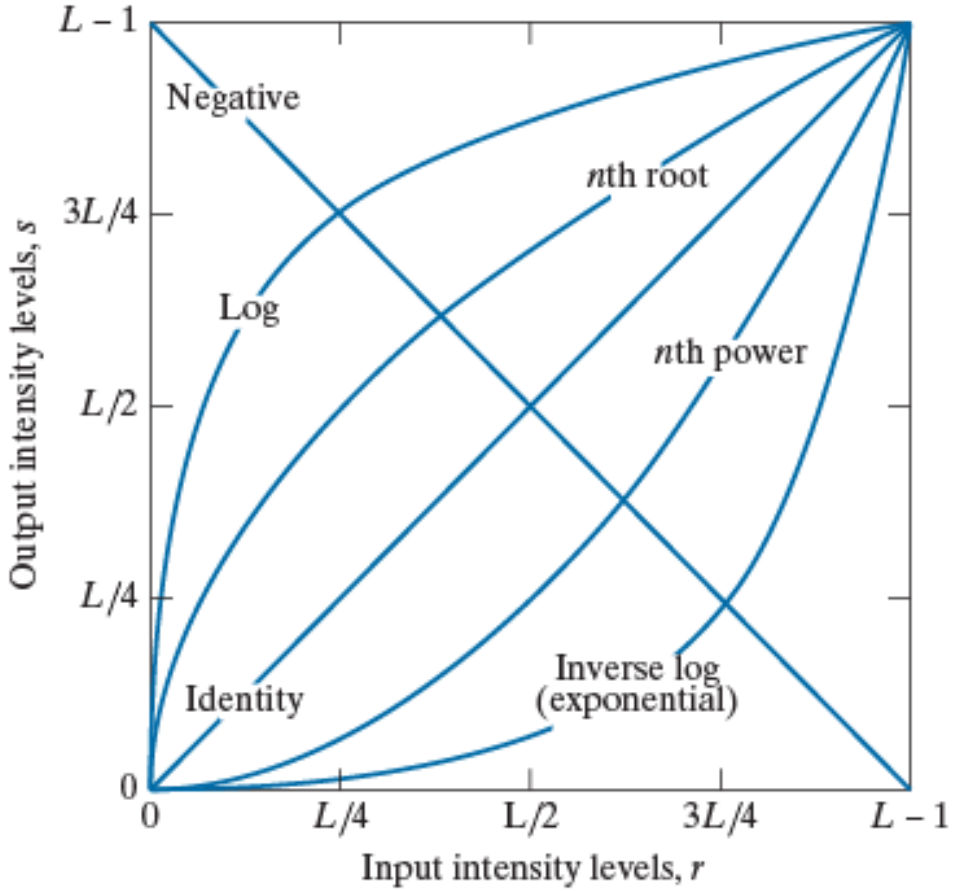
Contrast stretching function



Thresholding function

Intensity transformations

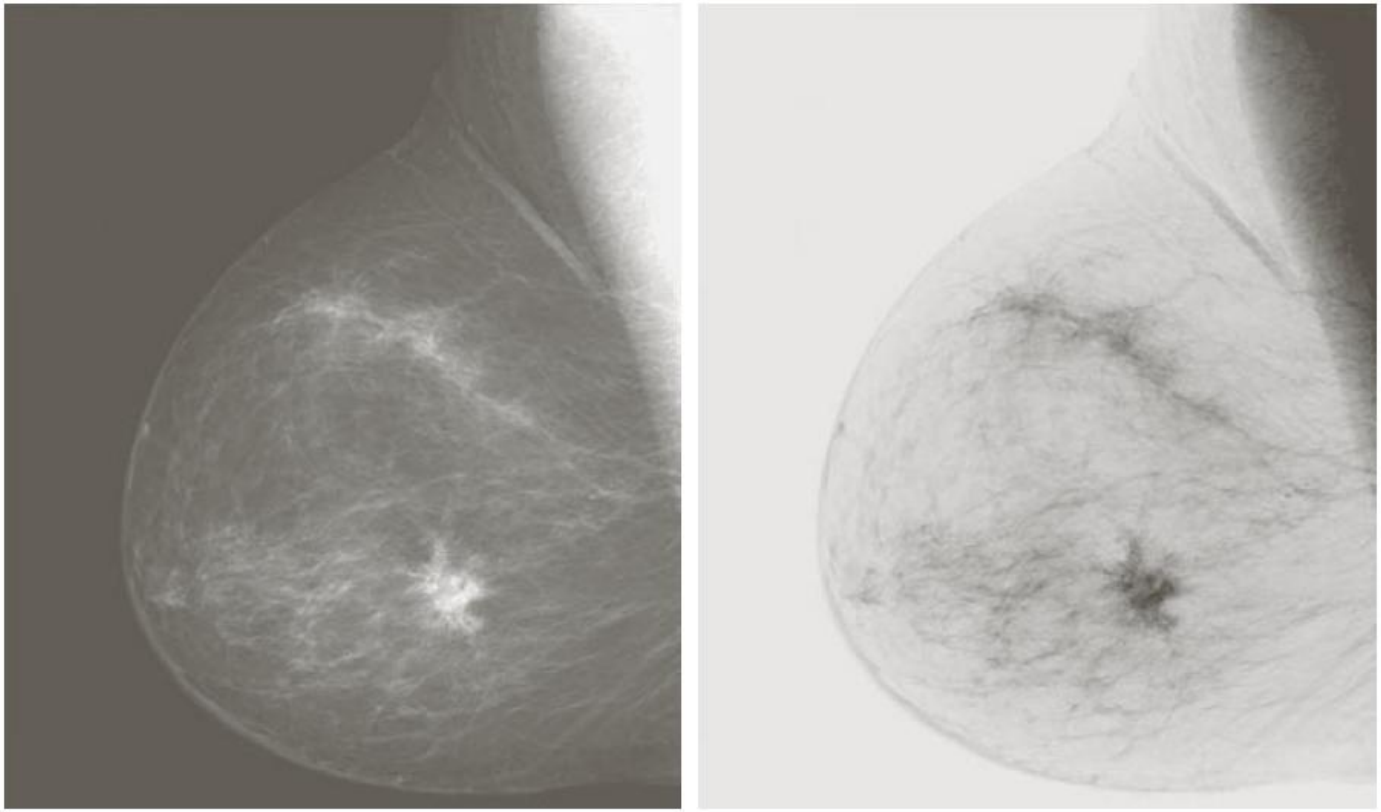
for 8 bits per pixel (bpp)/channel images, $L = 2^8 = 256$



Some basic transformation functions

Negative transformation

$$s = (L-1) - r, \text{ where } r \in [0, L-1]$$



a b

FIGURE 3.4 (a) A digital mammogram. (b) Negative image obtained using Eq. (3-3). (Image (a) Courtesy of General Electric Medical Systems.)

Gamma transformation

$$s = (L-1)c\left(\frac{r}{L-1}\right)^{\gamma}, \text{ where } r \in [0, L-1] \qquad c \text{ is often } 1$$

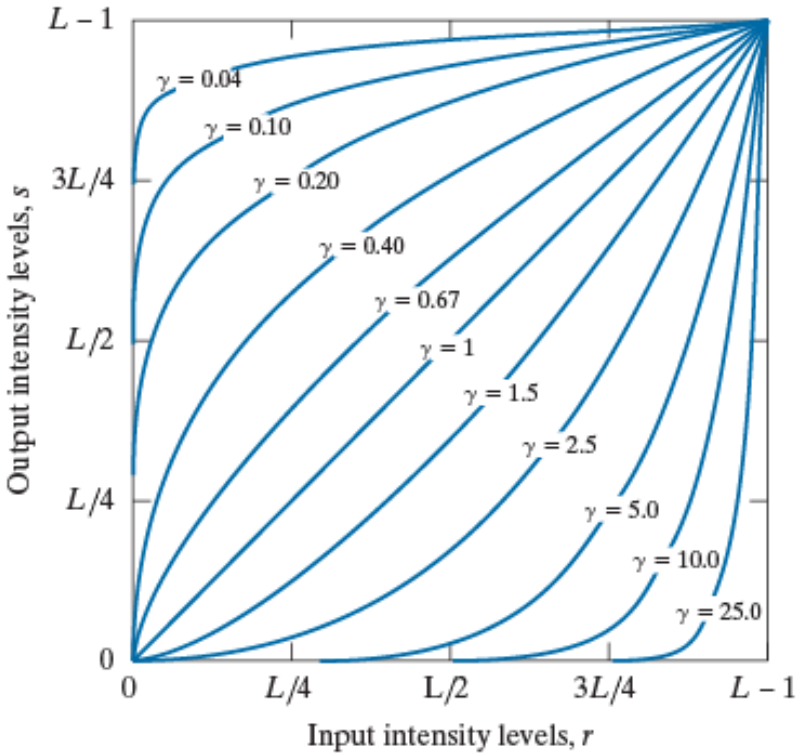
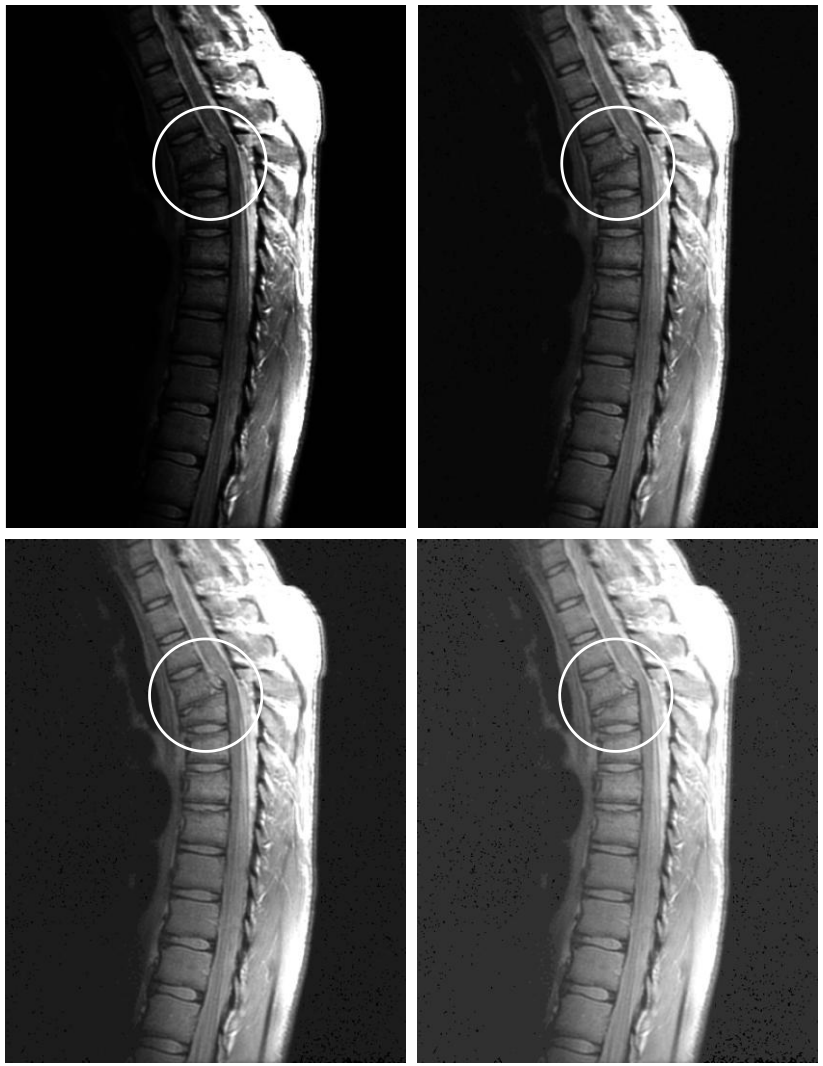


FIGURE 3.6
Plots of the gamma equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). Each curve was scaled *independently* so that all curves would fit in the same graph. Our interest here is on the *shapes* of the curves, not on their relative values.

Gamma transformation

Dark image

$\gamma < 1$



a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine (the region of the fracture is enclosed by the circle). (b)–(d) Results of applying the transformation in Eq. (3-5) with $c = 1$ and $\gamma = 0.6$, 0.4, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

$$s = cr^{\gamma}$$

Gamma transformation

Light image

$\gamma > 1$



a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3-5) with $\gamma = 3.0$, 4.0, and 5.0, respectively. ($c = 1$ in all cases.) (Original image courtesy of NASA.)

$s = cr^{\gamma}$

Piecewise-linear transformations

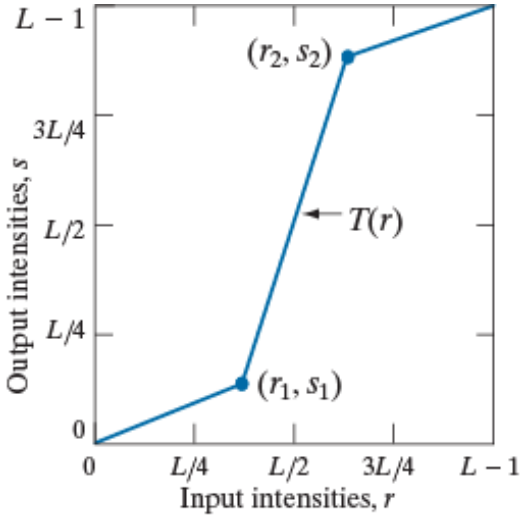
- Contrast stretching
- Intensity-level slicing
- Bit-plane slicing

Contrast stretching

A low-contrast electron microscope image of pollen, magnified 700 times



Input image



Piecewise linear transformation function

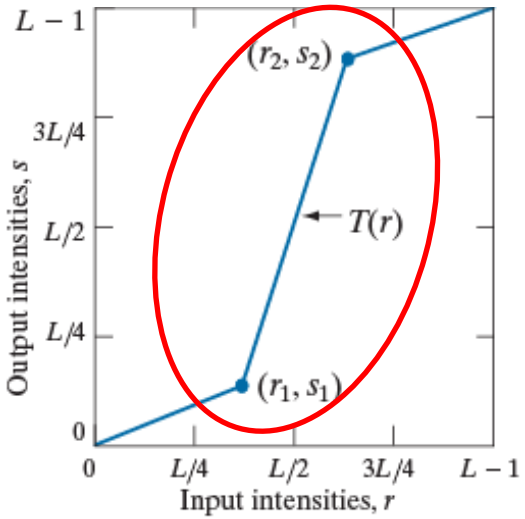


Output image

FIGURE 3.10
(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Contrast stretching

- Solve for the parameters of each piece



Piecewise linear function

$$r_1 \mapsto s_1$$
$$r_2 \mapsto s_2$$

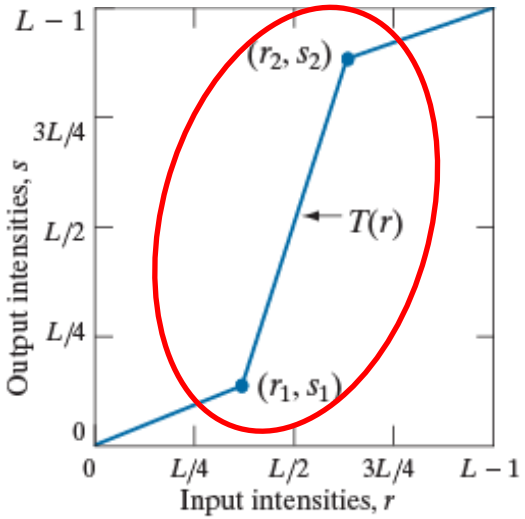
$$s_1 = ar_1 + b$$
$$s_2 = ar_2 + b$$

Solve for a and b

In matrix form

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} r_1 & 1 \\ r_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Contrast stretching



$$s_1 = ar_1 + b$$
$$s_2 = ar_2 + b$$

Solve for a and b

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} r_1 & 1 \\ r_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

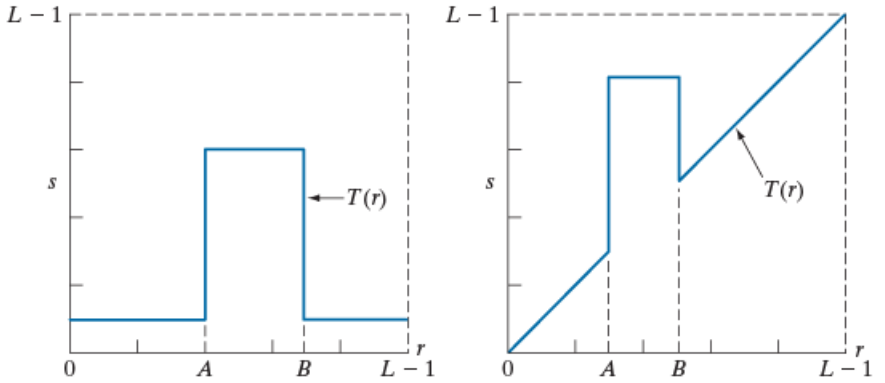
$$\begin{bmatrix} r_1 & 1 \\ r_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{1}{r_1 - r_2} \begin{bmatrix} 1 & -1 \\ -r_2 & r_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{1}{r_1 - r_2} \begin{bmatrix} s_1 - s_2 \\ r_1 s_2 - r_2 s_1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

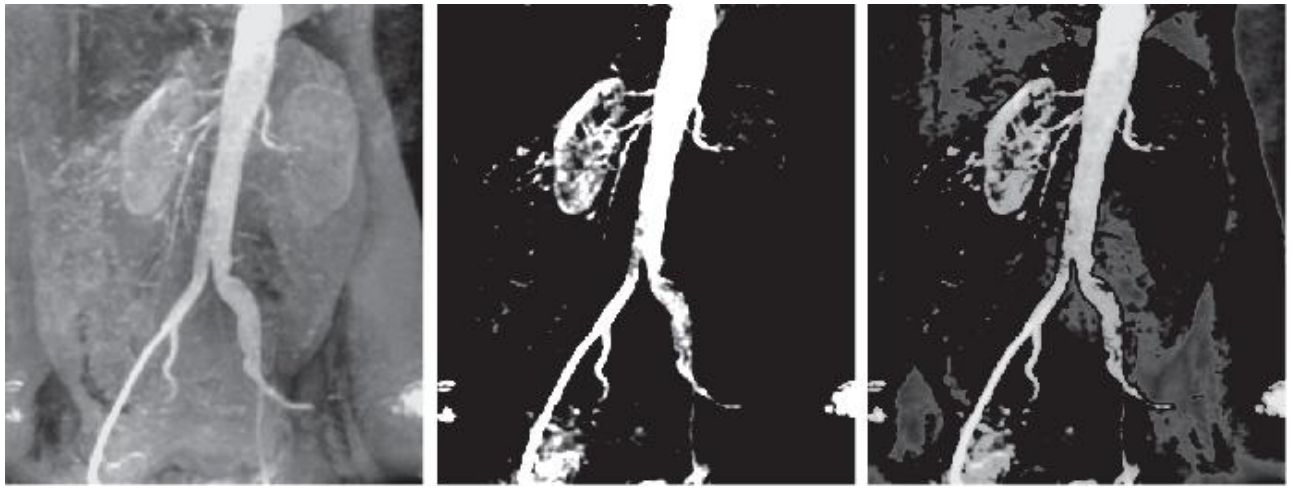
$$a = \frac{s_1 - s_2}{r_1 - r_2}$$
$$b = \frac{r_1 s_2 - r_2 s_1}{r_1 - r_2}$$

Intensity-level slicing



a b

FIGURE 3.11
(a) This transformation function highlights range $[A, B]$ and reduces all other intensities to a lower level. (b) This function highlights range $[A, B]$ and leaves other intensities unchanged.



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Bit-plane slicing

- Example: 8 bpp/channel images
  - Digits of binary number

$$b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0$$

Most significant bit Least significant bit

  - Binary to decimal

$$d = b_7 2^7 + b_6 2^6 + b_5 2^5 + b_4 2^4 + b_3 2^3 + b_2 2^2 + b_1 2^1 + b_0 2^0$$

Bit-plane slicing

- Example: 8 bpp/channel images

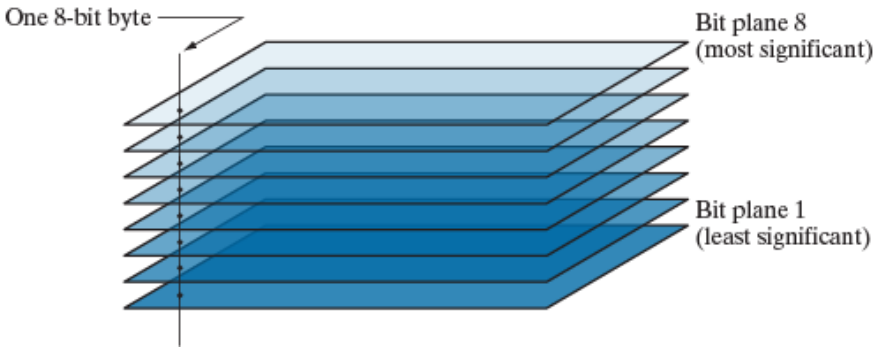


FIGURE 3.13
Bit-planes of an 8-bit image.

	Bit-plane	Binary	Hexadecimal
$[2^7, 2^8 - 1]$	8	$r\&10000000_b$	$r\&0x80$
	7	$r\&01000000_b$	$r\&0x40$
	6	$r\&00100000_b$	$r\&0x20$
	5	$r\&00010000_b$	$r\&0x10$
	4	$r\&00001000_b$	$r\&0x08$
	3	$r\&00000100_b$	$r\&0x04$
	2	$r\&00000010_b$	$r\&0x02$
	1	$r\&00000001_b$	$r\&0x01$

Bitwise AND

Bit-plane slicing

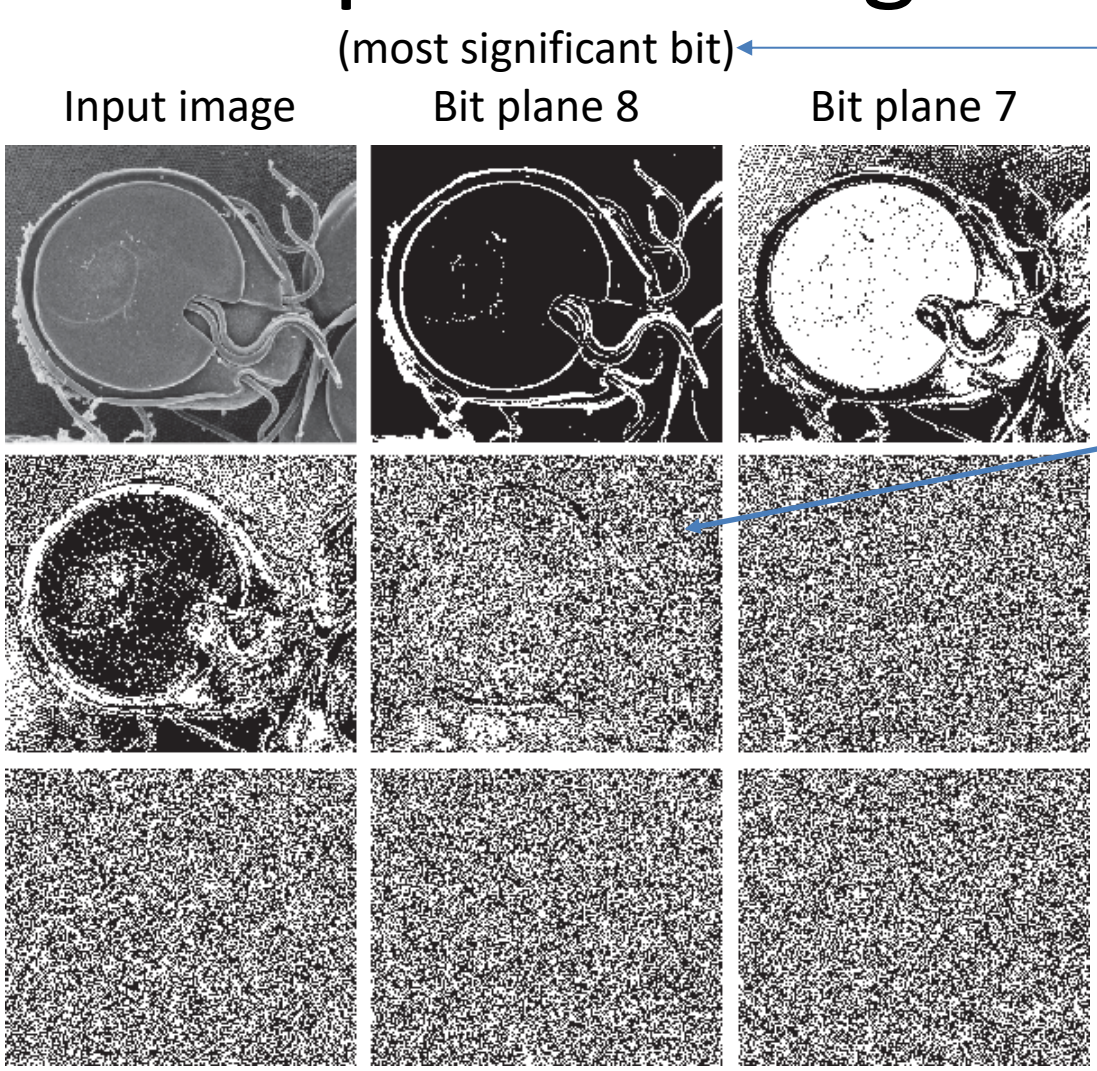
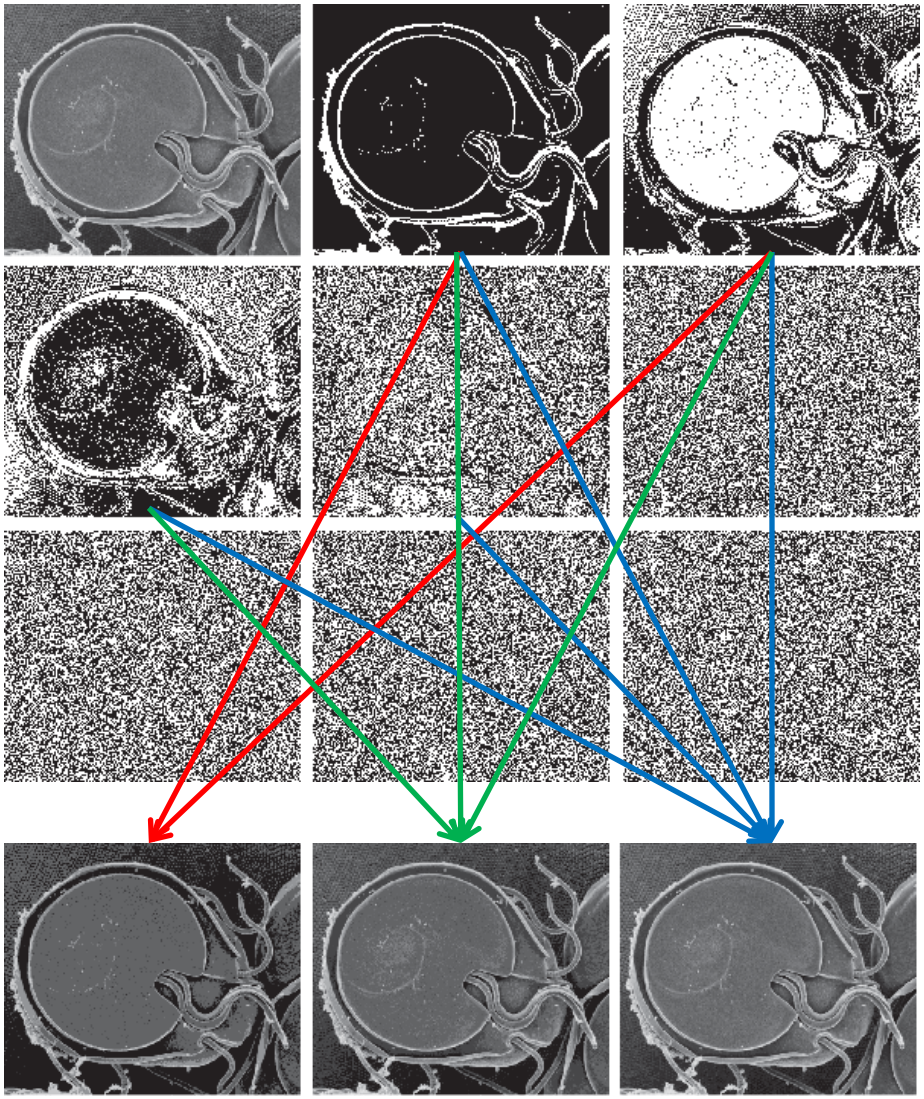


FIGURE 3.14
SEM image of a trophozoite that causes a disease called *giardiasis*. (Courtesy of Dr. Stan Erlandsen, U.S. Center for Disease Control and Prevention.)

Bit-plane slicing

Input image



Reconstructed images

Last before "noisy" bit planes

a b c

FIGURE 3.15 Image reconstructed from bit planes: (a) 8 and 7; (b) 8, 7, and 6; (c) 8, 7, 6, and 5.

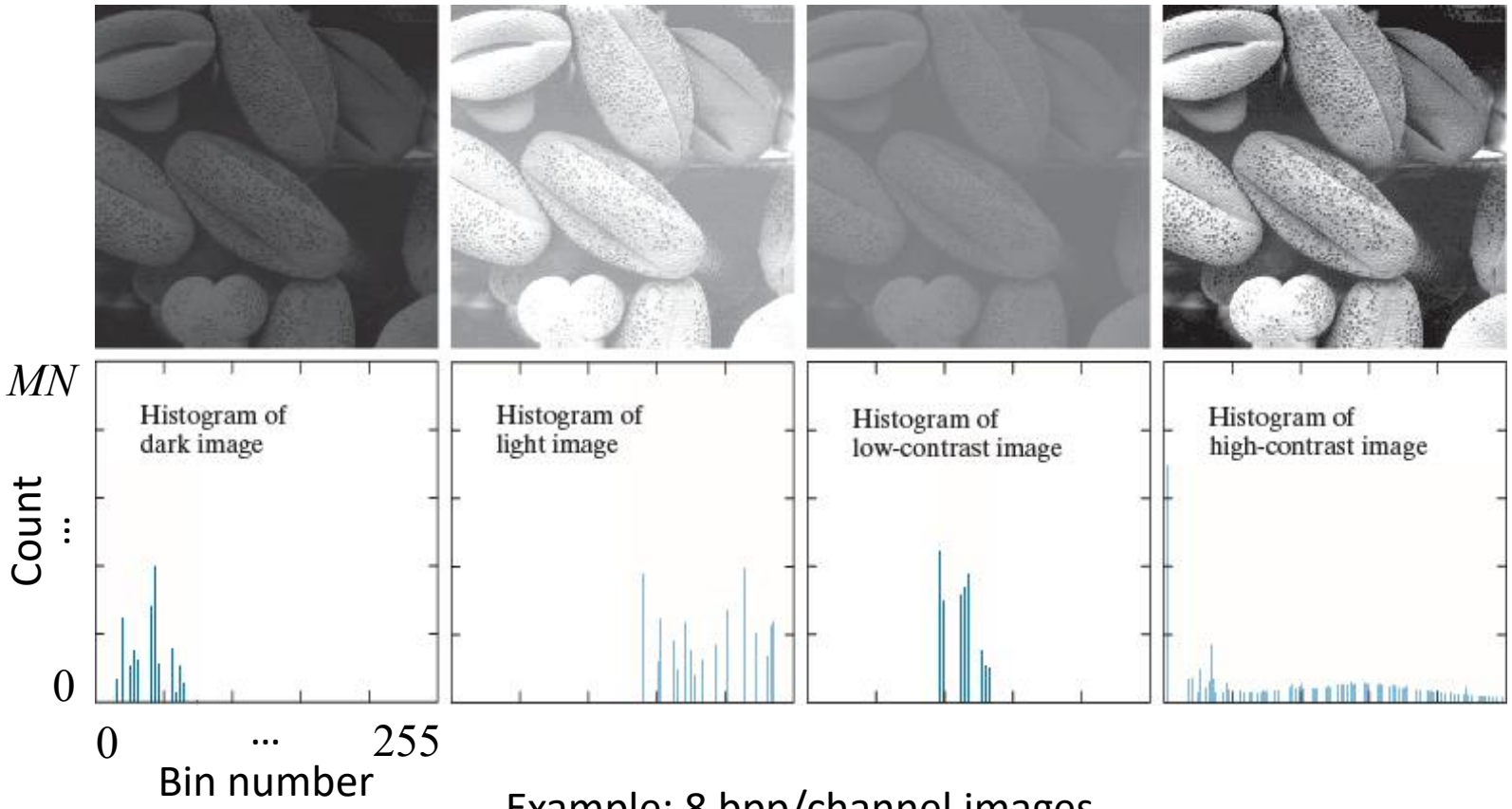
Histogram processing

- Histogram equalization
- Histogram matching
- Local histogram equalization

Histogram

a b c d

FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.



Example: 8 bpp/channel images

Normalized histogram

- Probability density function (pdf) $0 \leq p(x) \leq 1$
  - Area under curve is 1

Continuous

- Histogram
  - Area under curve is total number of pixels = MN
- Normalized histogram

$$\text{normalized histogram} = \frac{1}{MN}\text{histogram} \approx \text{pdf}$$

Discrete

$p_f(r)$ is approximate pdf of input image $f(x, y)$
$p_g(s)$ is approximate pdf of output image $g(x, y)$

Cumulative distribution function

- Cumulative distribution function

$$\mathrm{cdf}_f(r) = \int_0^r p_f(w)\mathrm{d}w$$

Continuous

$$\mathrm{cdf}_f(r) = \sum_{k=0}^{r} p_f(k)$$

Discrete

  - Monotonically increasing
  - Last value is maximum value = 1

$\mathrm{cdf}_f(r)$ is cdf of input image $f(x, y)$
$\mathrm{cdf}_g(s)$ is cdf of output image $g(x, y)$

Histogram equalization

- Conditions
  - Output intensity values will never be less than corresponding input values

    $$s = T(r) \text{ is monotonic increasing in } 0 \leq r \leq L-1$$

  - The range of output intensities is the same as the input

    $$0 \leq T(r) \leq L-1 \quad 0 \leq r \leq L-1$$

  - If we further require the inverse mapping from s back to r

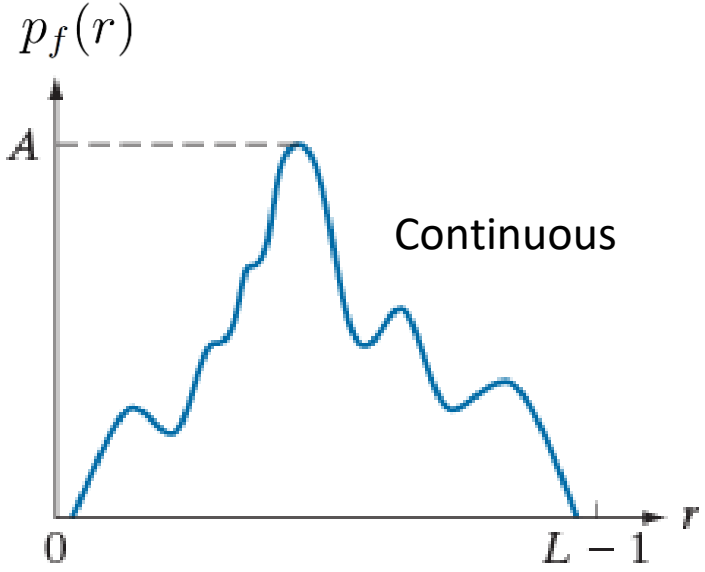
    $$r = T^{-1}(s) \quad 0 \leq s \leq L-1$$

    be one-to-one, then

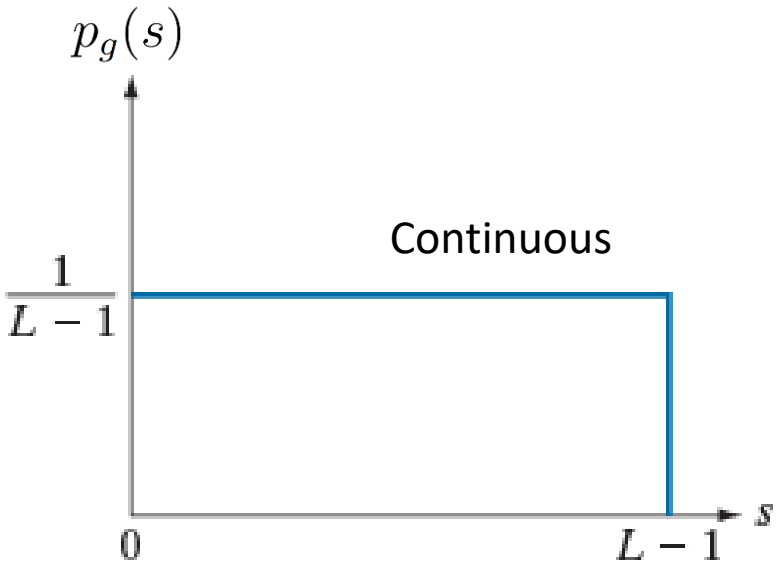
    $$s = T(r) \text{ is } \textit{strictly} \text{ monotonic increasing in } 0 \leq r \leq L-1$$

Histogram equalization

- Objective: after (forward) mapping, the output image pdf is flat



Input image pdf

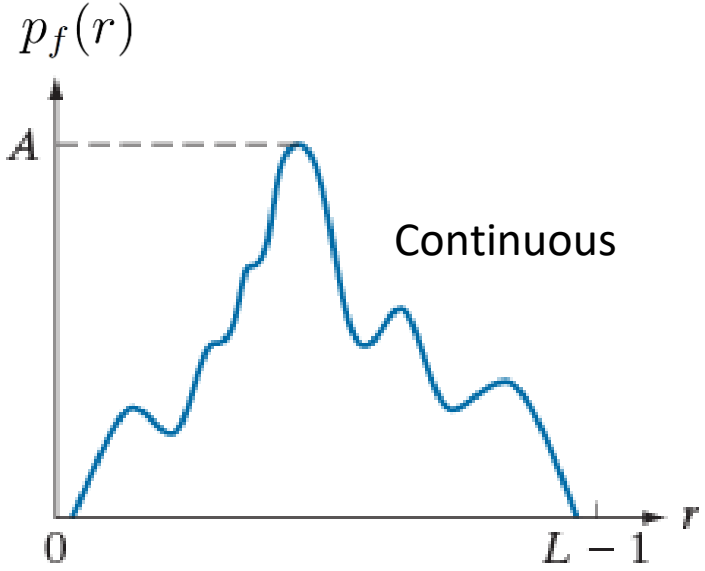


Output image pdf

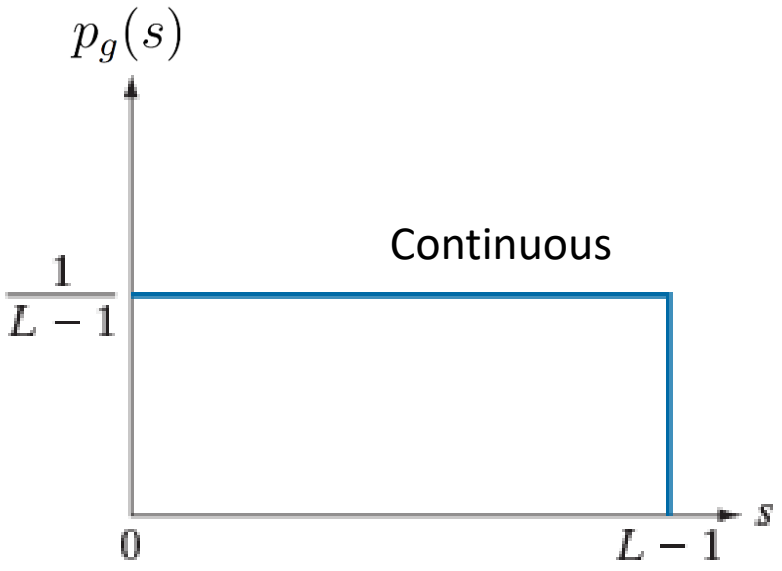
Histogram equalization

- This objective is satisfied by the histogram equalization intensity transformation

$$s = T(r) = (L-1)\int_0^r p_f(w)\mathrm{d}w$$ Continuous



Input image pdf



Output image pdf

Histogram equalization

Proof of $s = T(r) = (L-1)\int_0^r p_f(w)\mathrm{d}w$

From calculus, recall the derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit, i.e.,

$$\frac{\mathrm{d}s}{\mathrm{d}r} = \frac{\mathrm{d}T(r)}{\mathrm{d}r}$$

$$\frac{\mathrm{d}s}{\mathrm{d}r} = (L-1)\frac{\mathrm{d}}{\mathrm{d}r}\left[\int_0^r p_f(w)\mathrm{d}w\right]$$

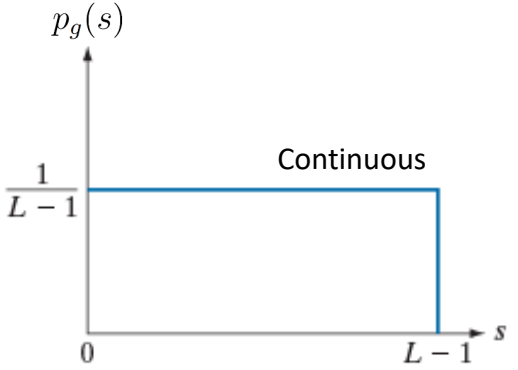
$$\frac{\mathrm{d}s}{\mathrm{d}r} = (L-1)p_f(r)$$

From probability theory, recall if $p_f(r)$ and $T(r)$ are known, and $T(r)$ is continuous and differentiable over the range of values of interest, then the pdf of the transformed (i.e., mapped) variable $s = T(r)$ can be obtained as

$$p_g(s) = p_f(r)\left|\frac{\mathrm{d}r}{\mathrm{d}s}\right|$$

$$p_g(s) = p_f(r)\left|\frac{1}{(L-1)p_f(r)}\right|$$

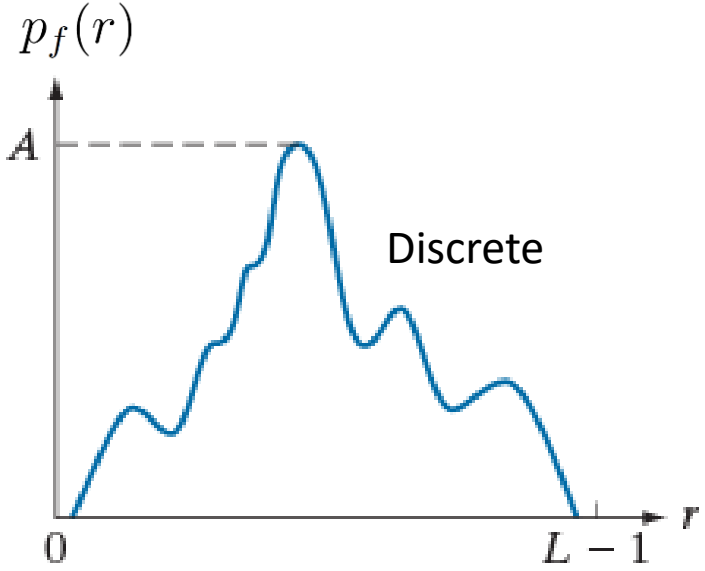
$$p_g(s) = \frac{1}{L-1} \quad 0 \le s \le L-1$$



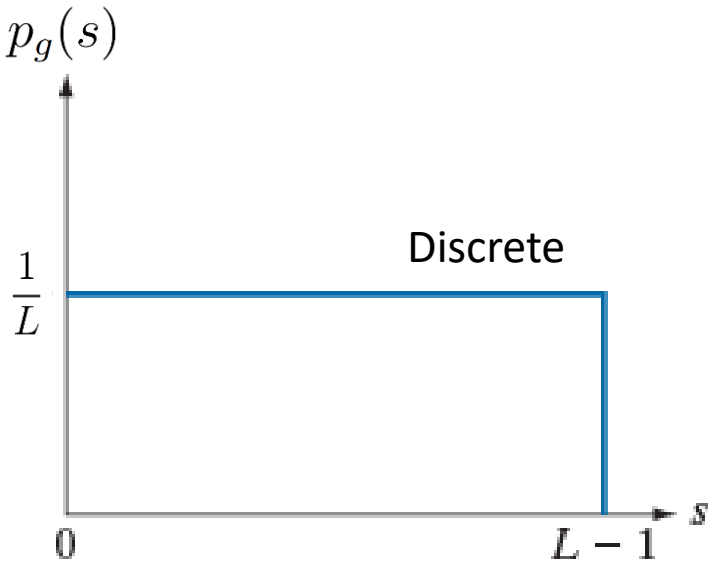
Output image pdf

Histogram equalization

- Objective: for **discrete** images, after (forward) mapping, **desired** output image approximate pdf is flat



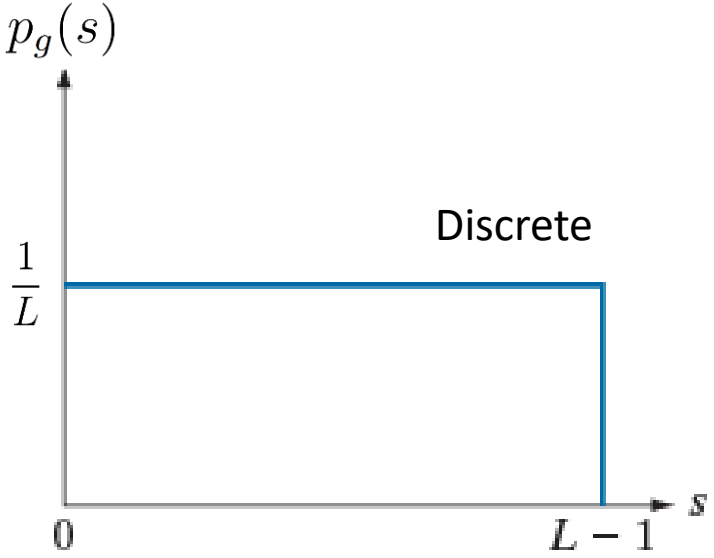
Input image
normalized histogram
(approximate pdf)



Desired output image
normalized histogram
(approximate pdf)

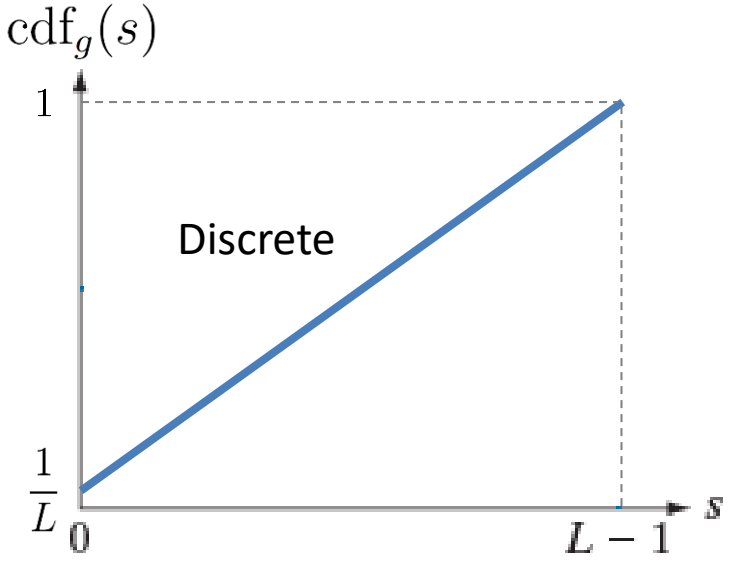
Histogram equalization

$$p_g(s) = \frac{1}{L} \quad s = 0, 1, 2, \ldots, L-1$$



Desired output image normalized histogram (approximate pdf)

$$\mathrm{cdf}_g(s) = \sum_{k=0}^{s} p_g(k) = \frac{i+1}{L} \quad s = 0, 1, 2, \ldots, L-1$$

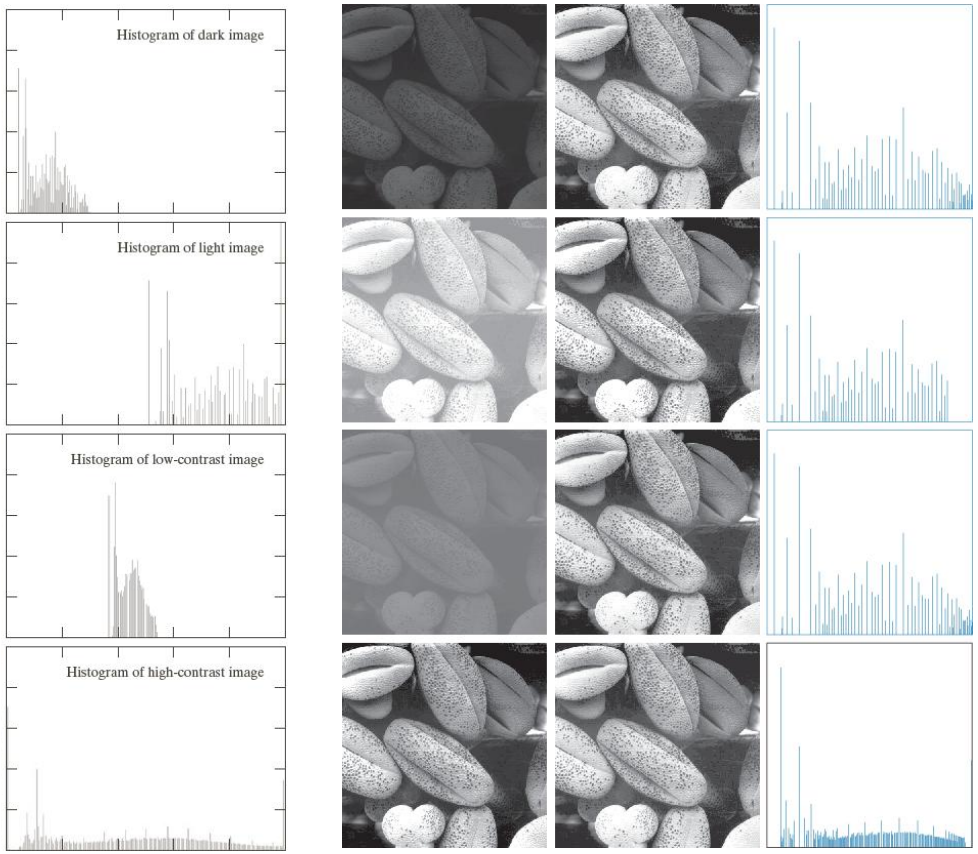


Desired output image cdf

Histogram equalization

Discrete

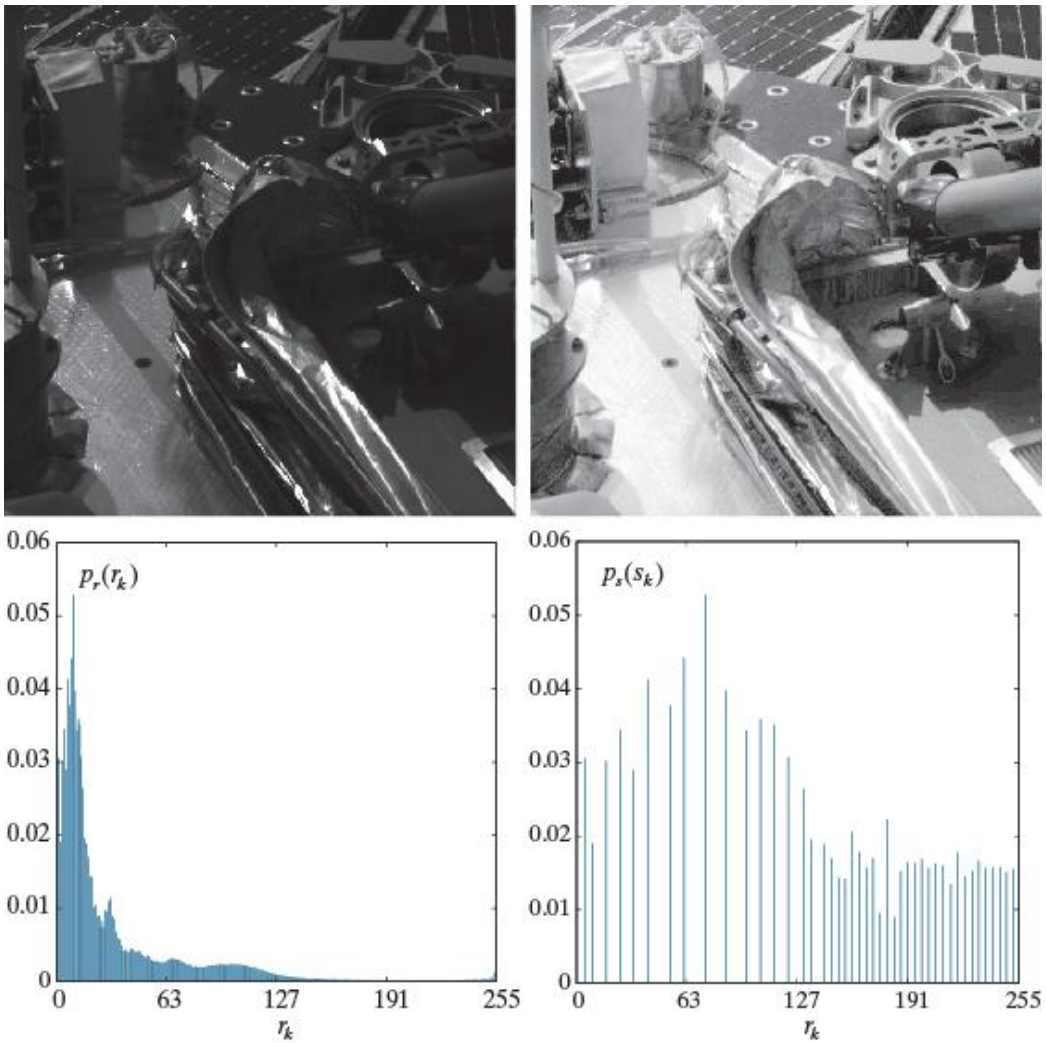
$$s = T(r) = (L-1)\mathrm{cdf}_f(r) \quad r = 0, 1, 2, \ldots, L-1$$



In general, discrete pdf will not be flat

FIGURE 3.20 Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

Histogram equalization



a b
c d

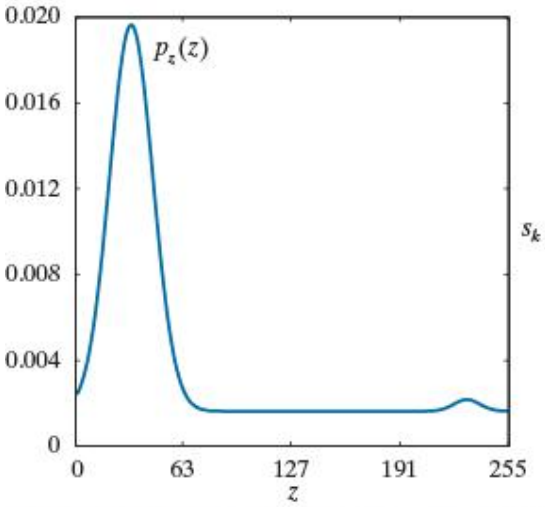
FIGURE 3.22
(a) Image from Phoenix Lander. (b) Result of histogram equalization. (c) Histogram of image (a). (d) Histogram of image (b). (Original image courtesy of NASA.)

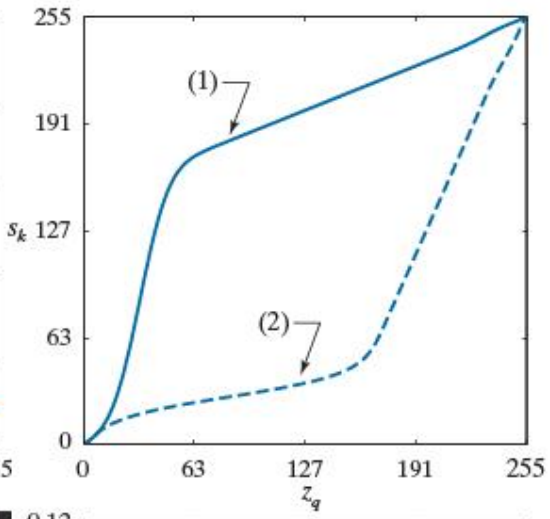
In general, discrete pdf will not be flat

Example: 8 bpp/channel images

Histogram matching

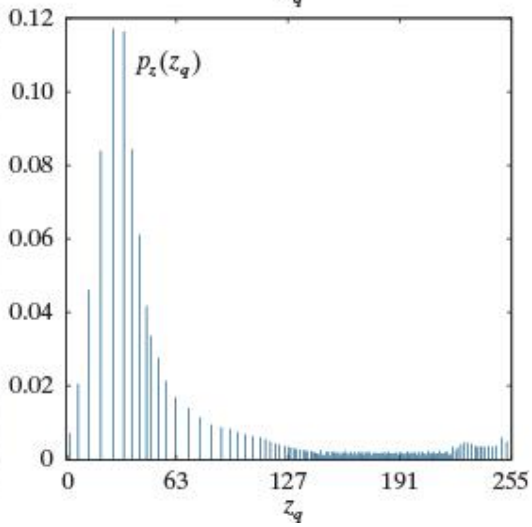
Specified histogram to match





Intensity transformation





Histogram of intensity transformed image

a b
c d

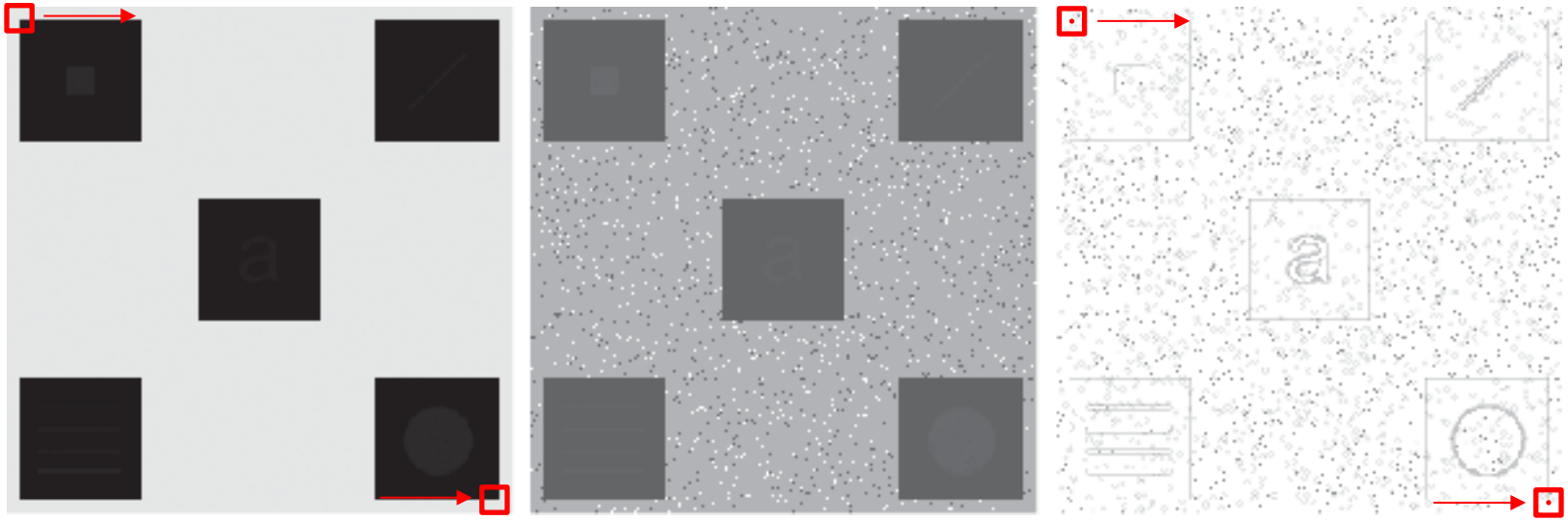
FIGURE 3.26 Histogram specification. (a) Specified histogram. (b) Transformation $G(z_q)$, labeled (1), and $G^{-1}(s_k)$, labeled (2). (c) Result of histogram specification. (d) Histogram of image (c).

Local histogram equalization

Input image

Global

Local



Neighborhoods

a b c

FIGURE 3.32 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization.

Next Lecture

- Spatial filtering
- Reading
  - Chapter 3: Intensity Transformations and Spatial Filtering
    - Sections 3.4, 3.5, 3.6, and 3.8