Feature Extraction (Part 1)

Image Processing

CSE 166

Lecture 16

Announcements

- Assignment 6 is due Jun 3, 11:59 PM
- Final exam is Jun 12, 7:00 PM-9:59 PM
- Please complete TA and course evaluations
- Reading
  - Chapter 12: Feature extraction
    - Sections 12.1, 12.4, and 12.6

Feature extraction

- Feature extraction is comprised of
  - Feature detection
  - Feature description
    - A feature descriptor is
      - **Invariant** with respect to a set of transformations if its value remains unchanged after the application of any transformation from the family
      - **Covariant** with respect to a set of transformations if applying any transformation from the set produces the same result in the descriptor

Features

- Features
  - Local (a member of a set)
  - Global (the entire set)
- Categories
  - Boundaries (not covered in CSE 166)
  - Regions
  - Whole images

Region feature descriptors

- Basic descriptors

Perimeter p
Area A
Compactness

$$\text{compactness} = \frac{p^2}{A}$$

Circularity

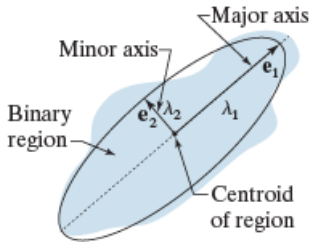
$$\text{cicularity} = \frac{4\pi A}{p^2}$$

Effective diameter (i.e., diameter of a circle with same area)

$$d_e = 2\sqrt{\frac{A}{\pi}}$$

Region feature descriptors

- Basic descriptors



Eccentricity

$$\text{eccentricity} = \sqrt{1 - \frac{\lambda_2}{\lambda_1}}$$

where $\sqrt{\lambda_1}$ is the semimajor axis length and $\sqrt{\lambda_2}$ is the semiminor axis length of the ellipse. λ_1 and λ_2 are calculated as the major eigenvalue and minor eigenvalue, respectively, of the covariance matrix

$$\Sigma_{\mathbf{z}} = \frac{1}{K-1}\sum_{k=1}^{K}(\mathbf{z}_k - \boldsymbol{\mu}_{\mathbf{z}})(\mathbf{z}_k - \boldsymbol{\mu}_{\mathbf{z}})^{\top}$$

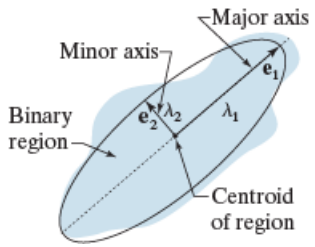
where

$\mathbf{z}_k = (x_k, y_k)^{\top}$	Point (pixel coordinates)
$\boldsymbol{\mu}_{\mathbf{z}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{z}_k$	Mean vector (i.e., centroid)
K	Numer of points (pixels)

Region feature descriptors

The eigen decomposition of the covariance matrix $\Sigma_{\mathbf{z}}$ results in the eigenvalues λ_1 and λ_2, and their associated orthonormal eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively.

$$
\begin{aligned}
(\Sigma_{\mathbf{z}} - \lambda_i \mathtt{I})\mathbf{v}_i &= \mathbf{0} \text{ for } i = 1, 2, \text{ where } \det(\Sigma_{\mathbf{z}} - \lambda_i \mathtt{I}) = 0 \\
\Sigma_{\mathbf{z}}\mathbf{v}_i - \lambda_i \mathbf{v}_i &= \mathbf{0} \text{ for } i = 1, 2 \\
\Sigma_{\mathbf{z}} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} - \begin{bmatrix} \lambda_1 \mathbf{v}_1 & \lambda_2 \mathbf{v}_2 \end{bmatrix} &= \mathbf{0} \\
\Sigma_{\mathbf{z}} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \operatorname{diag}(\lambda_1, \lambda_2) &= \mathbf{0} \\
\Sigma_{\mathbf{z}} \mathtt{V} - \mathtt{V}\Lambda &= \mathbf{0}, \text{ where } \mathtt{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \text{ and } \Lambda = \operatorname{diag}(\lambda_1, \lambda_2) \\
\Sigma_{\mathbf{z}} \mathtt{V} &= \mathtt{V}\Lambda \\
\Sigma_{\mathbf{z}} &= \mathtt{V}\Lambda\mathtt{V}^{\top}
\end{aligned}
$$



Feature space

- *n*-dimensional feature vector
  - "Packaged" form of features
  - Example
    - Compactness, circularity, and eccentricity form a 3-vector

a b c d

FIGURE 12.22 Compactness, circularity, and eccentricity of some simple binary regions.

Descriptor				
Compactness	10.1701	42.2442	15.9836	13.2308
Circularity	1.2356	0.2975	0.7862	0.9478
Eccentricity	0.0411	0.0636	0	0.8117

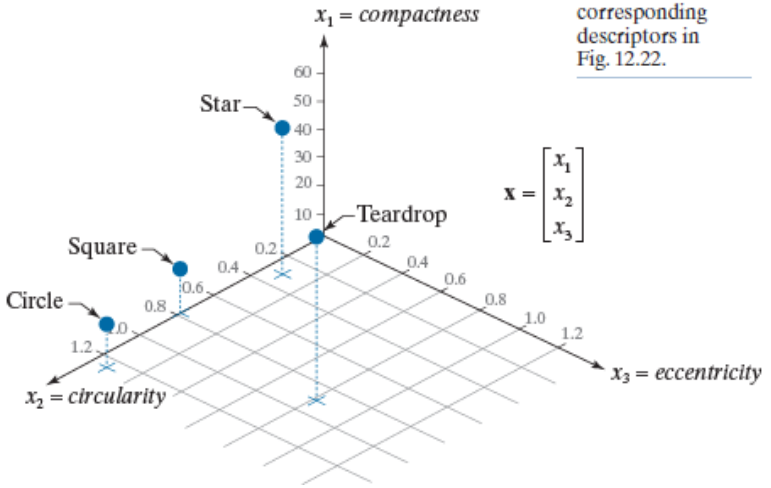


FIGURE 12.23 The descriptors from Fig. 12.22 in 3-D feature space. Each dot shown corresponds to a feature vector whose components are the three corresponding descriptors in Fig. 12.22.

Texture as a region descriptor

No information regarding spatial relationships between pixels

- Statistical texture measures
  - Statistical moments
    - The *n*-th moment of *z* about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

where the mean

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

$p(z_i)$ is the normalized histogram and L is the number of gray levels

      - The second, third, and fourth moments are the variance, skewness, and kurtosis, respectively

Texture as a region descriptor

No information regarding spatial relationships between pixels

- Statistical texture measures
  - Relative intensity smoothness

$$R(z) = 1 - \frac{1}{1 + \sigma^2(z)}$$

  - Uniformity

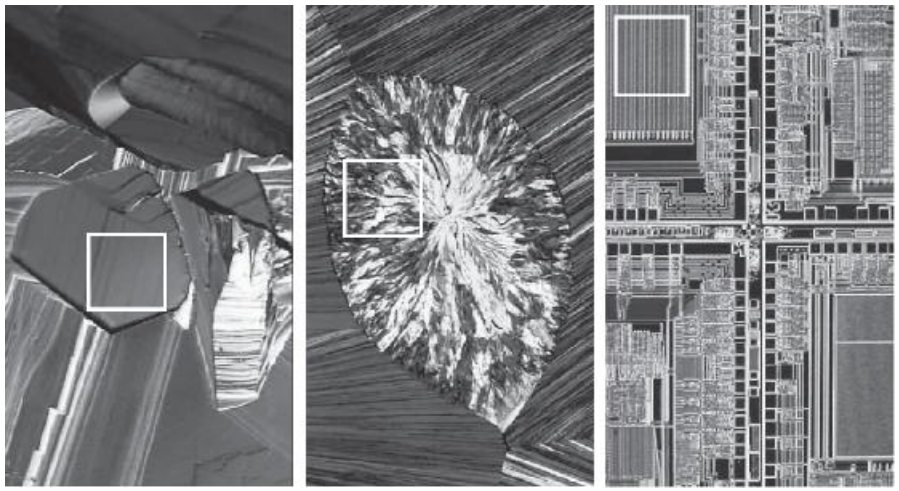
$$U(z) = \sum_{k=0}^{L-1} p^2(z_i)$$

  - Average entropy

$$e(z) = -\sum_{k=0}^{L-1} p(z_i) \log_2(p(z_i))$$

Statistical texture measures

Smooth Coarse Regular



a b c

FIGURE 12.29 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

TABLE 12.2
Statistical texture measures for the subimages in Fig. 12.29.

Texture	Mean	Standard deviation	*R* (normalized)	3rd moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

Texture as a region descriptor

- Graylevel co-occurrence matrix

Relative positions of pixels in an image

  - Let *Q* be an operator that defines the position of two pixels relative to each other in image *f* with number of gray levels *L*
  - Let **G** be a matrix whose element g_{ij} is the number of times that pixel pairs with intensities z_i and z_j occur in image *f* in the position specified by *Q*
    - Probability a pair of points satisfy *Q* is

$$p_{ij} = \frac{g_{ij}}{n}$$

where *n* is the number of pixels pairs that satisfy *Q* (i.e., *n* is the sum of the elements in *Q*)

Graylevel co-occurrence matrix

- Number of gray levels $L = 8$
- Position operator Q is "one pixel immediately to the right"

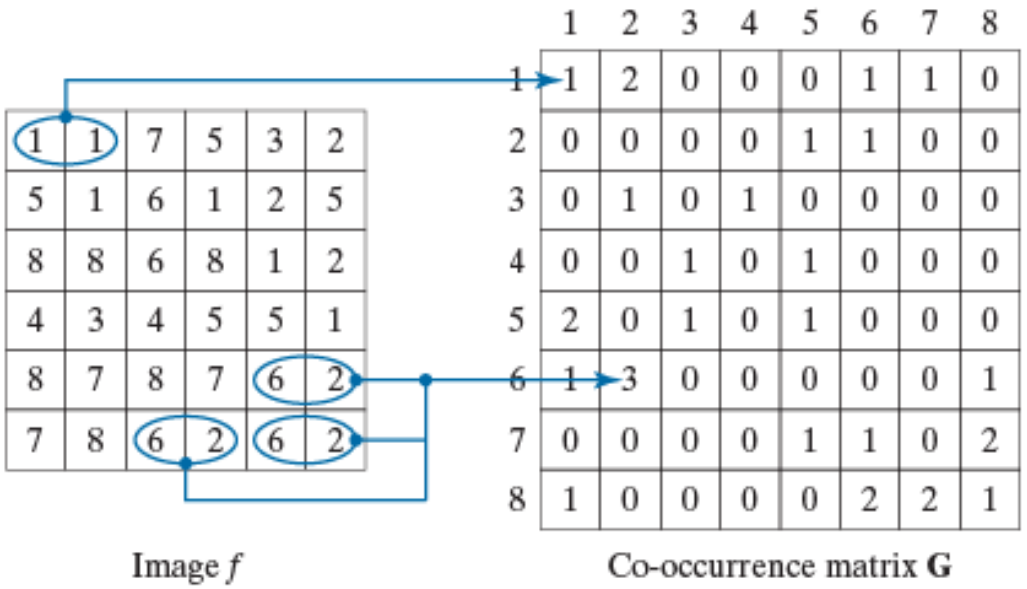
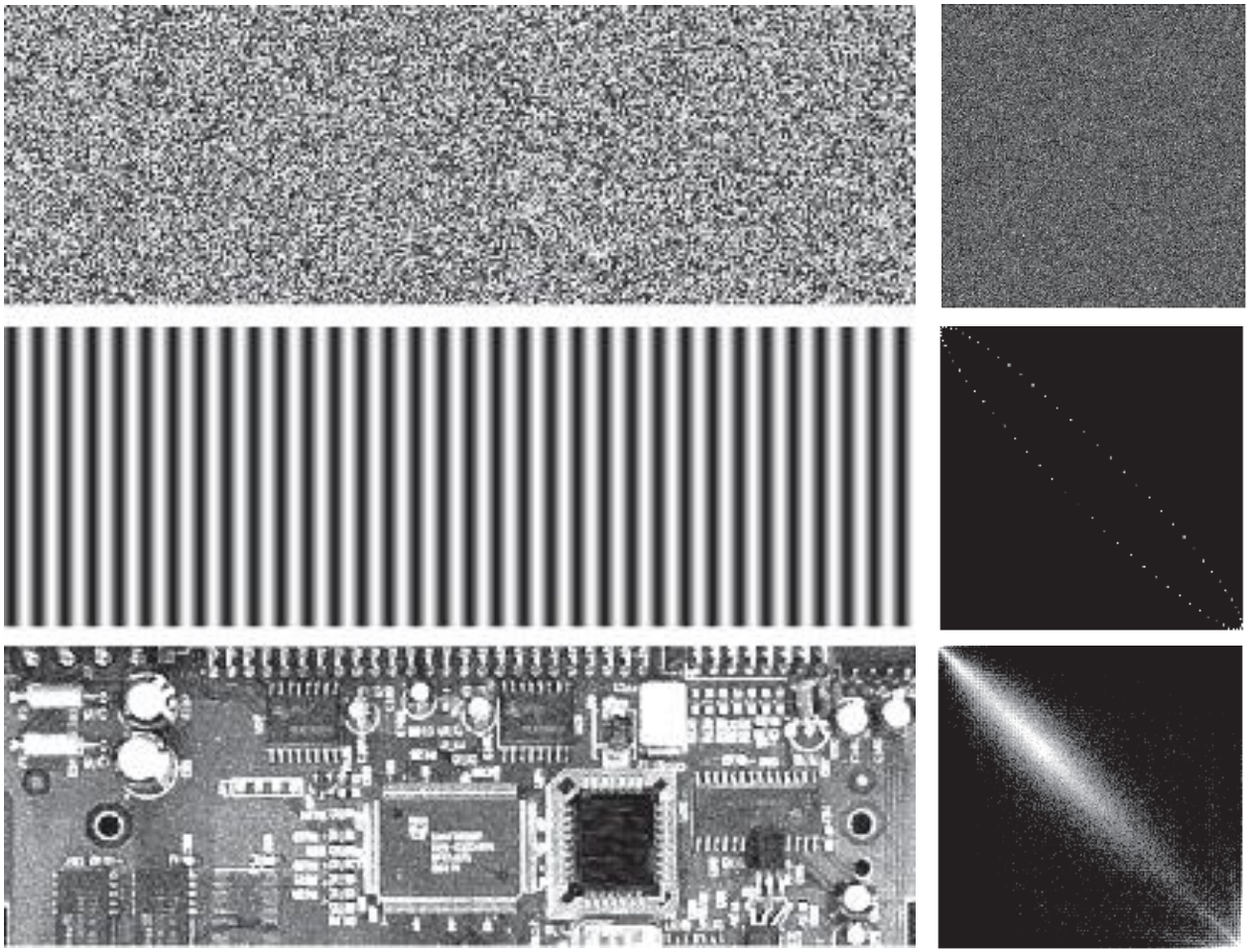


FIGURE 12.30 How to construct a co-occurrence matrix.

Graylevel co-occurrence matrix

Images



Graylevel co-occurrence matrices

a
b
c

FIGURE 12.31 Images whose pixels have (a) random, (b) periodic, and (c) mixed texture patterns. Each image is of size 263×800 pixels.

Texture as a region descriptor

- Graylevel co-occurrence matrix-based feature descriptors

TABLE 12.3 Descriptors used for characterizing co-occurrence matrices of size $K \times K$. The term p_{ij} is the ij-th term of $\mathbf{G}$ divided by the sum of the elements of $\mathbf{G}$.

Descriptor	Explanation	Formula
Maximum probability	Measures the strongest response of $\mathbf{G}$. The range of values is [0, 1].	$\max_{i,j}(p_{ij})$
Correlation	A measure of how correlated a pixel is to its neighbor over the entire image. The range of values is 1 to –1 corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i-m_r)(j-m_c)p_{ij}}{\sigma_r\,\sigma_c}$ $\sigma_r \neq 0;\ \sigma_c \neq 0$
Contrast	A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when $\mathbf{G}$ is constant) to $(K-1)^2$.	$\sum_{i=1}^{K}\sum_{j=1}^{K}(i-j)^2 p_{ij}$
Uniformity (also called Energy)	A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image.	$\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}^2$
Homogeneity	Measures the spatial closeness to the diagonal of the distribution of elements in $\mathbf{G}$. The range of values is [0, 1], with the maximum being achieved when $\mathbf{G}$ is a diagonal matrix.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{p_{ij}}{1+\lvert i-j\rvert}$
Entropy	Measures the randomness of the elements of $\mathbf{G}$. The entropy is 0 when all p_{ij}'s are 0, and is maximum when the p_{ij}'s are uniformly distributed. The maximum value is thus $2\log_2 K$.	$-\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}\log_2 p_{ij}$

where

$$m_r = \sum_{i=1}^{K} i \sum_{j=1}^{K} p_{ij}$$

$$m_c = \sum_{j=1}^{K} j \sum_{i=1}^{K} p_{ij}$$

$$\sigma_r^2 = \sum_{i=1}^{K}(i-m_r)^2 \sum_{j=1}^{K} p_{ij}$$

$$\sigma_c^2 = \sum_{j=1}^{K}(j-m_c)^2 \sum_{i=1}^{K} p_{ij}$$

Graylevel co-occurrence matrix-based feature descriptors

a b c

FIGURE 12.32 256×256 co-occurrence matrices $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 12.31.



$\mathbf{G}_1$ $\mathbf{G}_2$ $\mathbf{G}_3$

TABLE 12.4
Descriptors evaluated using the co-occurrence matrices displayed as images in Fig. 12.32.

Normalized Co-occurrence Matrix	Maximum Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
$\mathbf{G}_1/n_1$	0.00006	–0.0005	10838	0.00002	0.0366	15.75
$\mathbf{G}_2/n_2$	0.01500	0.9650	00570	0.01230	0.0824	06.43
$\mathbf{G}_3/n_3$	0.06860	0.8798	01356	0.00480	0.2048	13.58

Whole-image features

- Feature descriptors applicable to entire images that are members of a large family of images
- Detection
  - Corner-like features
  - Regions
    - Could be described using the descriptors covered earlier

Detection of corner-like features

- Corner
  - A rapid change of direction in a curve
  - A highly effective feature
    - Distinctive
    - Reasonably invariant to viewpoint

Detection of corner-like features

- Examine a small window over an image

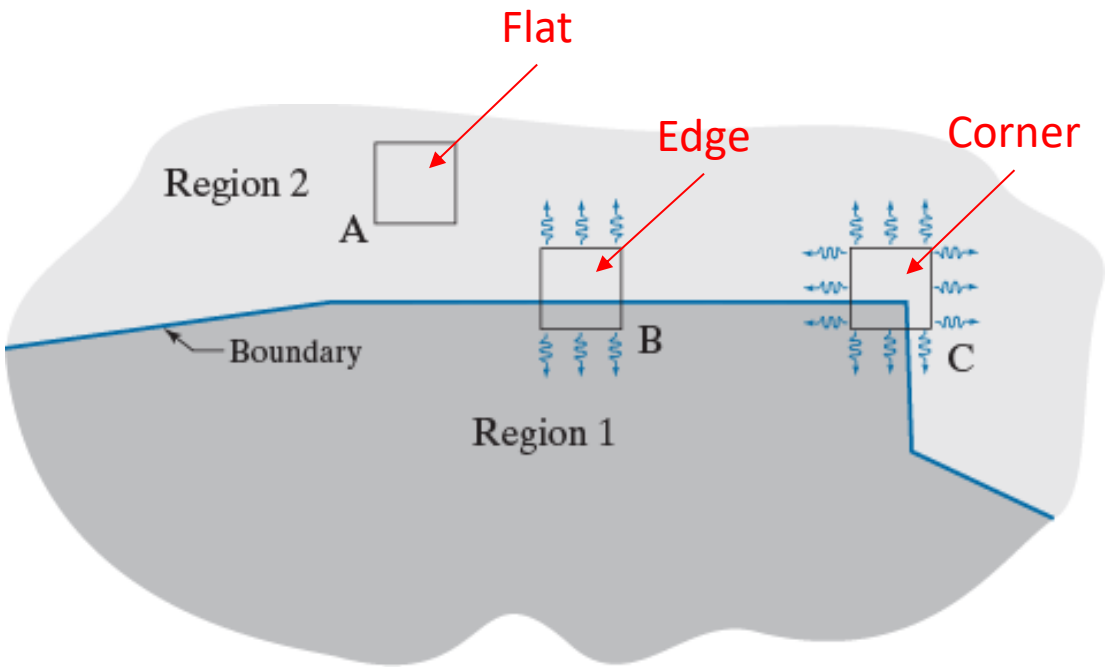


FIGURE 12.45 Illustration of how the Harris-Stephens corner detector operates in the three types of sub-regions indicated by A (flat), B (edge), and C (corner). The wiggly arrows indicate graphically a directional response in the detector as it moves in the three areas shown.

Detection of corner-like features

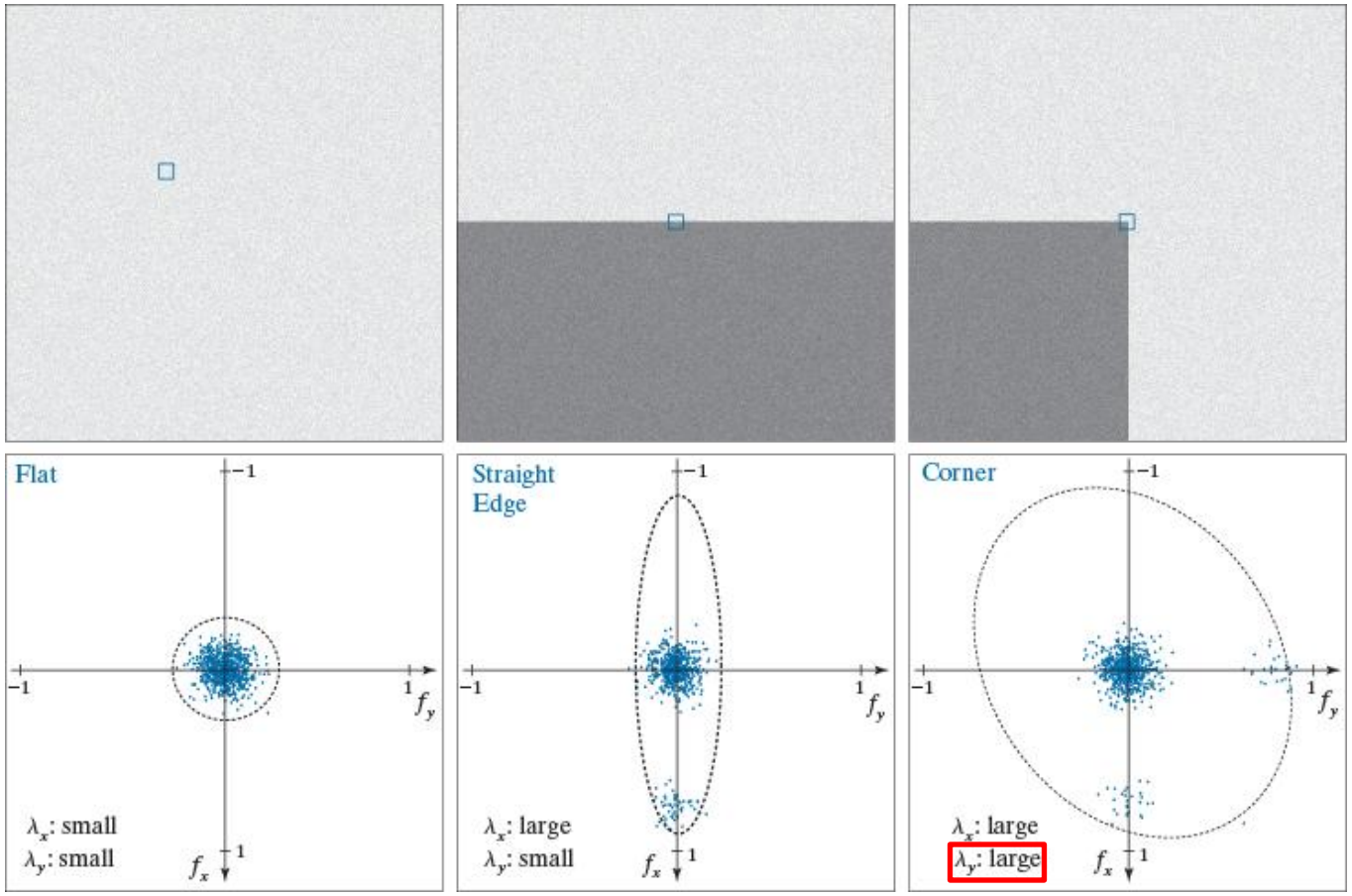
- For each window location, compute the spatial gradient matrix

$$\mathtt{M} = \begin{bmatrix} \sum_s \sum_t f_x^2(s,t) & \sum_s \sum_t f_x(s,t) f_y(s,t) \\ \sum_s \sum_t f_x(s,t) f_y(s,t) & \sum_s \sum_t f_y^2(s,t) \end{bmatrix}$$

  where f_x is the gradient in the *x*-direction and f_y is the gradient in the *y*-direction

- Then, compute eigenvalues of spatial gradient matrix

Eigenvalues of spatial gradient matrix



a b c
d e f

FIGURE 12.46 (a)–(c) Noisy images and image patches (small squares) encompassing image regions similar in content to those in Fig. 12.45. (d)–(f) Plots of value pairs (f_x, f_y) showing the characteristics of the eigenvalues of **M** that are useful for detecting the presence of a corner in an image patch.

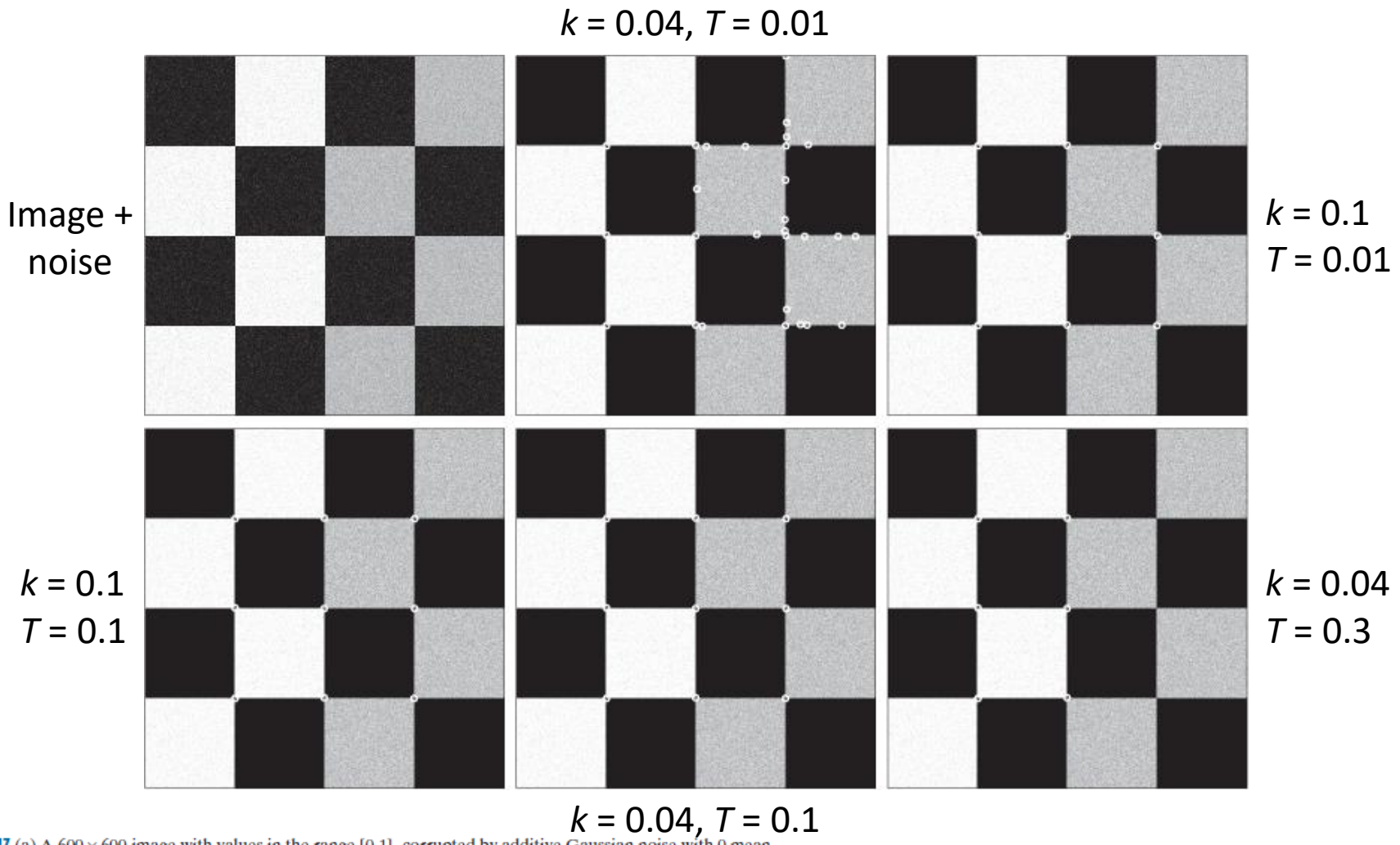
Detection of corner-like features

- Harris-Stephens corner detector
  - Run a small window over an image and compute spatial gradient matrix **M**
  - Approximate the minor eigenvalue of **M** to compute measurement image *R*
    - Include sensitivity factor *k*

$$R(x, y) = \det(\mathtt{M}) - k\,\mathrm{Tr}^2(\mathtt{M})$$

  - Threshold measurement image *R* using global threshold *T*
    - Corner at coordinates corresponding to $R > T$

Harris-Stephens corner detector

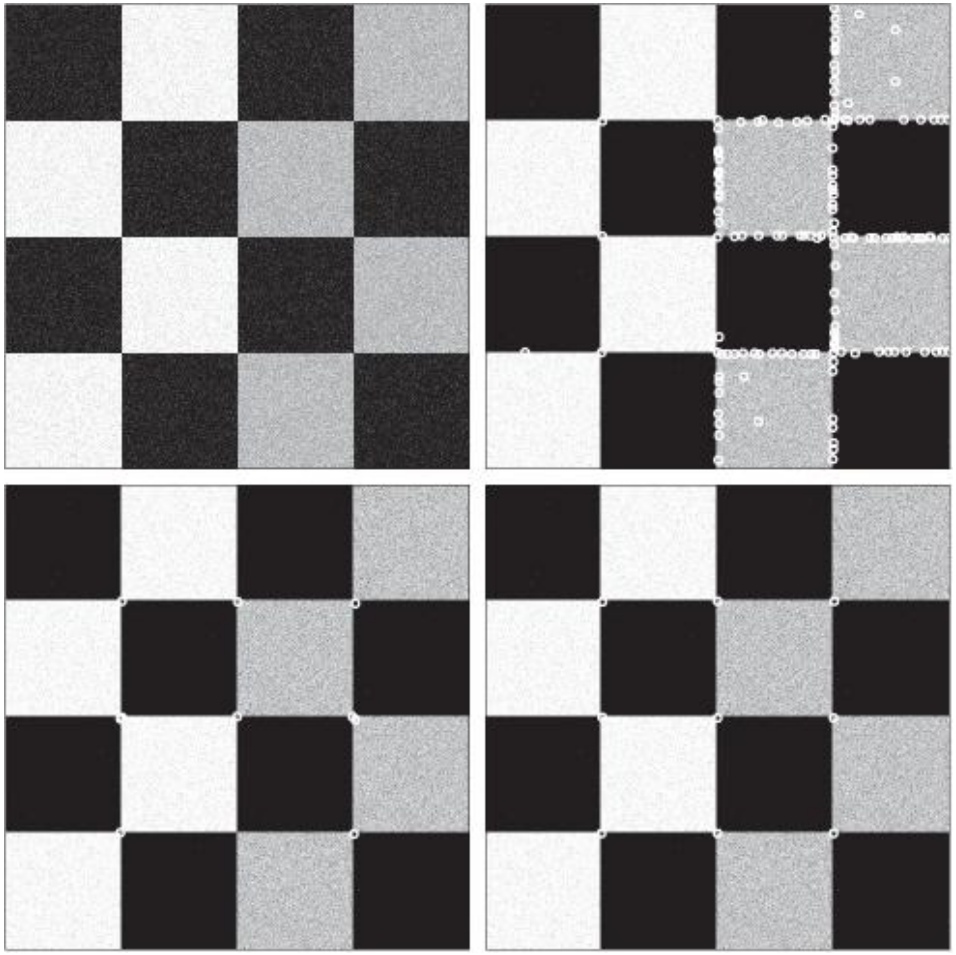


a b c
d e f

FIGURE 12.47 (a) A 600×600 image with values in the range [0,1], corrupted by additive Gaussian noise with 0 mean and variance of 0.006. (b) Result of applying the HS corner detector with $k = 0.04$ and $T = 0.01$ (the defaults). Several errors are visible. (c) Result using $k = 0.1$ and $T = 0.01$. (d) Result using $k = 0.1$ and $T = 0.1$. (e) Result using $k = 0.04$ and $T = 0.1$. (f) Result using $k = 0.04$ and $T = 0.3$ (only the strongest corners on the left were detected).

Harris-Stephens corner detector

Image +
more noise



$k = 0.04$
$T = 0.01$

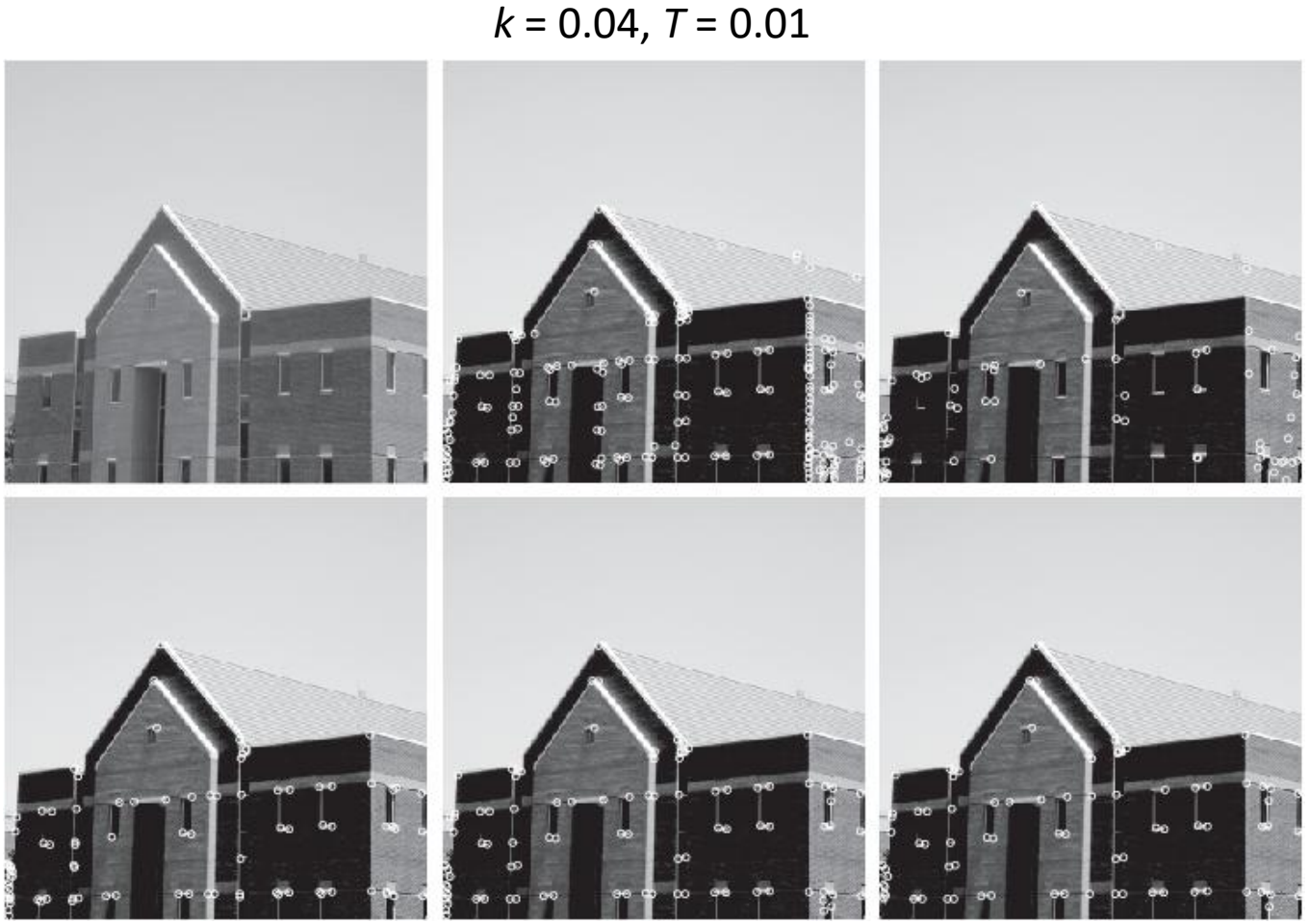
$k = 0.249$
$T = 0.01$

$k = 0.04$
$T = 0.15$

a b
c d

FIGURE 12.48
(a) Same as Fig. 12.47(a), but corrupted with Gaussian noise of mean 0 and variance 0.01. (b) Result of using the HS detector with $k = 0.04$ and $T = 0.01$ [compare with Fig. 12.47(b)]. (c) Result with $k = 0.249$, (near the highest value in our implementation), and $T = 0.01$. (d) Result of using $k = 0.04$ and $T = 0.15$.

Harris-Stephens corner detector



a b c
d e f

FIGURE 12.49 600 × 600 image of a building. (b) Result of applying the HS corner detector with $k = 0.04$ and $T = 0.01$ (the default values in our implementation). Numerous irrelevant corners were detected. (c) Result using $k = 0.249$ and the default value for T. (d) Result using $k = 0.17$ and $T = 0.05$. (e) Result using the default values of k and $T = 0.05$. (f) Result using the default value of k and $T = 0.07$.

Harris-Stephens corner detector

Image



$k = 0.04$
$T = 0.07$

Partial rotation invariance

Image rotated 5 degrees



$k = 0.04$
$T = 0.07$

a b

FIGURE 12.50 (a) Image rotated 5°. (b) Corners detected using the parameters used to obtain Fig. 12.49(f).

Detection of windows containing corner-like features

- Shi-Tomasi corner detector
  1. Run a small window over an image and compute spatial gradient matrix

$$\mathtt{M} = \begin{bmatrix} \sum_x \sum_y I_x(x,y) I_x(x,y) & \sum_x \sum_y I_x(x,y) I_y(x,y) \\ \sum_x \sum_y I_x(x,y) I_y(x,y) & \sum_x \sum_y I_y(x,y) I_y(x,y) \end{bmatrix}$$

$$\text{where } I_x = \frac{\partial I}{\partial x} \text{ and } I_y = \frac{\partial I}{\partial y}$$

Each element of matrix is a sum over a (square) window centered at pixel

Detection of windows containing corner-like features

- Shi-Tomasi corner detector
  2. Compute the minor eigenvalue of $\mathbf{M}$ to compute measurement image R

$$\lambda_{\min} = \frac{\operatorname{Tr}(\mathtt{M}) - \sqrt{\operatorname{Tr}(\mathtt{M})^2 - 4\det(\mathtt{M})}}{2}$$

  - The gradient matrix is positive semidefinite, so will not have negative eigenvalues. However, due to floating-point error, the expression under the square root may have very slightly negative values. To mitigate this, use

$$\lambda_{\min} = \frac{\operatorname{Tr}(\mathtt{M}) - \sqrt{\max(0, \operatorname{Tr}(\mathtt{M})^2 - 4\det(\mathtt{M}))}}{2}$$

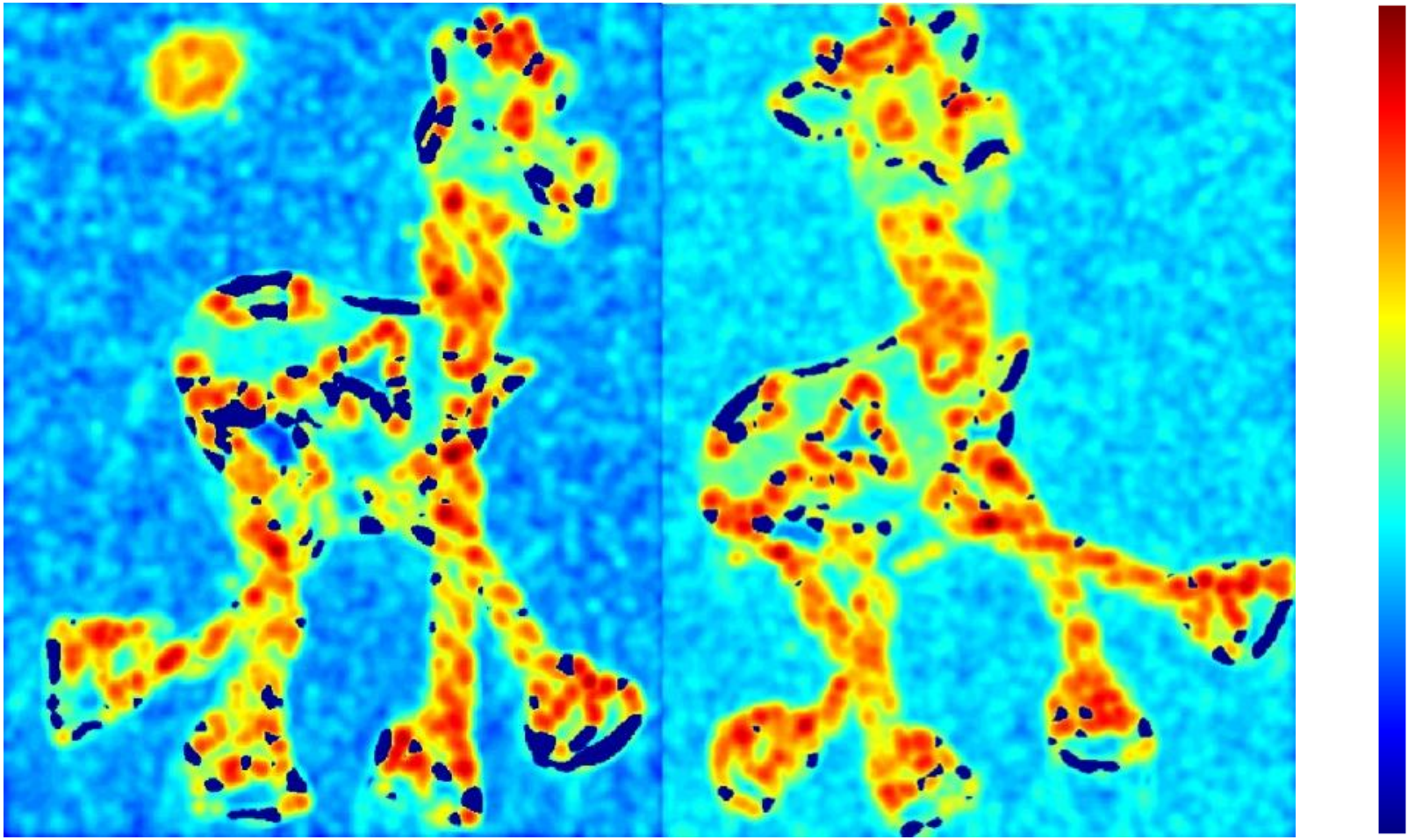
Input images



http://vims.cis.udel.edu/~chandra/

Measurement images

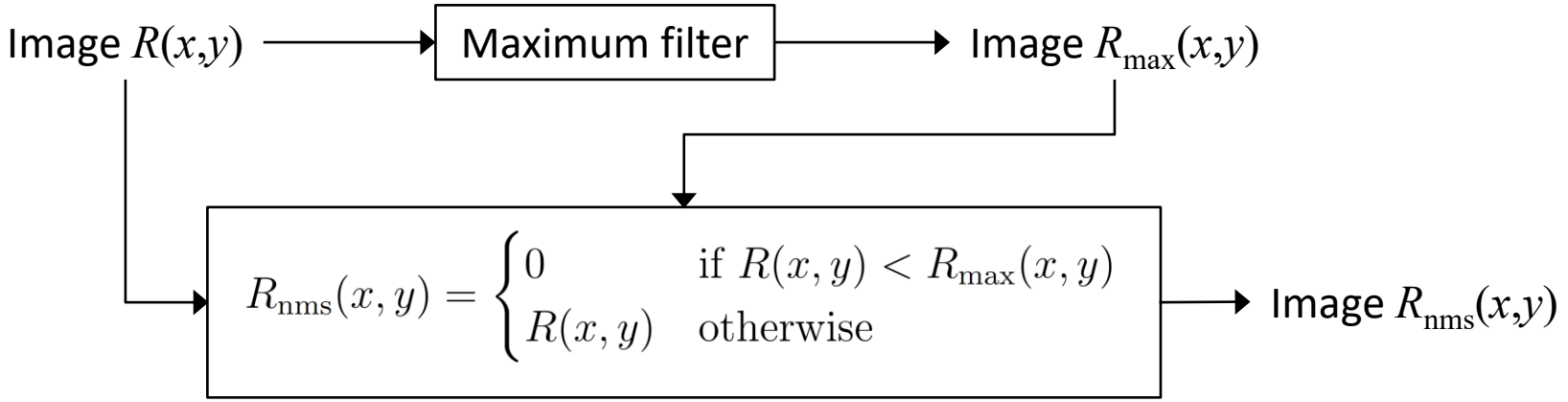
Pseudocolored



http://vims.cis.udel.edu/~chandra/

Detection of windows containing corner-like features

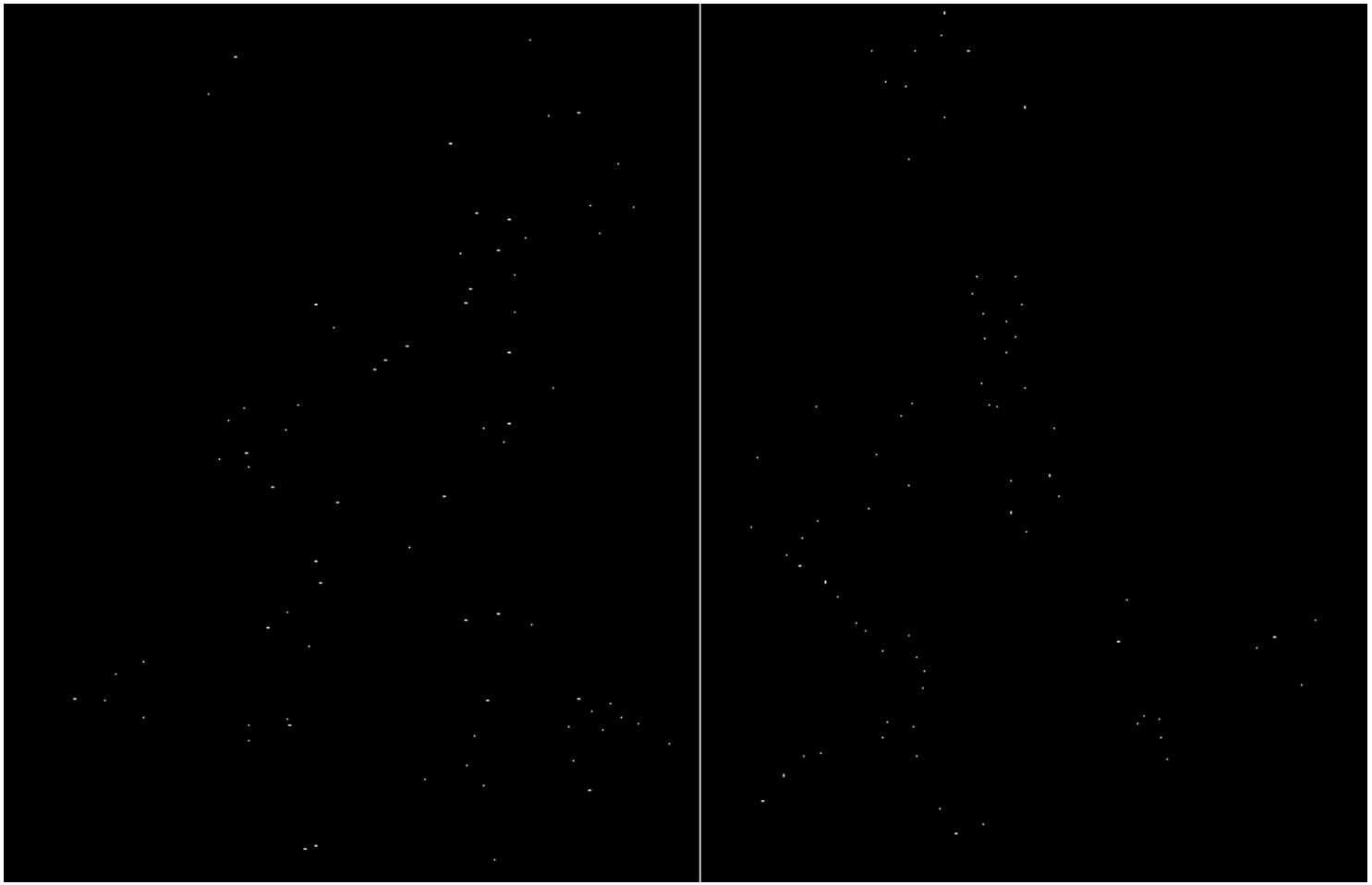
- Shi-Tomasi corner detector
  3. Apply nonmaximal suppression to the measurement image R
     - Prevents corners from being too close to each other



Detection of windows containing corner-like features

- Shi-Tomasi corner detector
  4. Threshold resulting image R_{nms} using global threshold T
     - Center of windows containing corner-like features are at coordinates corresponding to $R_{\text{nms}} > T$
       - Find coordinates of pixels in image $R_{\text{nms+thresh}} = R_{\text{nms}} > T$ with nonzero values

Local maxima above threshold



http://vims.cis.udel.edu/~chandra/

Center of windows containing corner-like features



http://vims.cis.udel.edu/~chandra/

Next Lecture

- Feature extraction
- Reading
  - Chapter 12: Feature extraction
    - Section 12.7