Edge Detection

Image Processing

CSE 166

Lecture 14

Announcements

- Assignment 5 is due today, 11:59 PM
- Assignment 6 will be released May 27
  - Due Jun 3, 11:59 PM
- Reading
  - Chapter 10: Image segmentation I: edge detection, thresholding, and region detection
    - Sections 10.1, 10.2, and 10.3

Image segmentation

R represents the entire spatial region occupied by an image

- $\cup_{i=1}^{n} R_i = R$
- R_i is a connected (e.g., 4-connected, 8-connected) set, for $i = 1, 2, \ldots, n$
- $R_i \cap R_j = \varnothing \, \forall \, i$ and j, for $i \neq j$, and disjoint R_i and R_j
- $Q(R_i) =$ TRUE for $i = 1, 2, \ldots, n$ (i.e., pixels in R_i have the same properties)
- $Q(R_i \cup R_j) =$ FALSE for any adjacent regions R_i and R_j

where $Q(R_k)$ is a predicate (e.g., same gray level) defined over the points in set R_k

Image segmentation

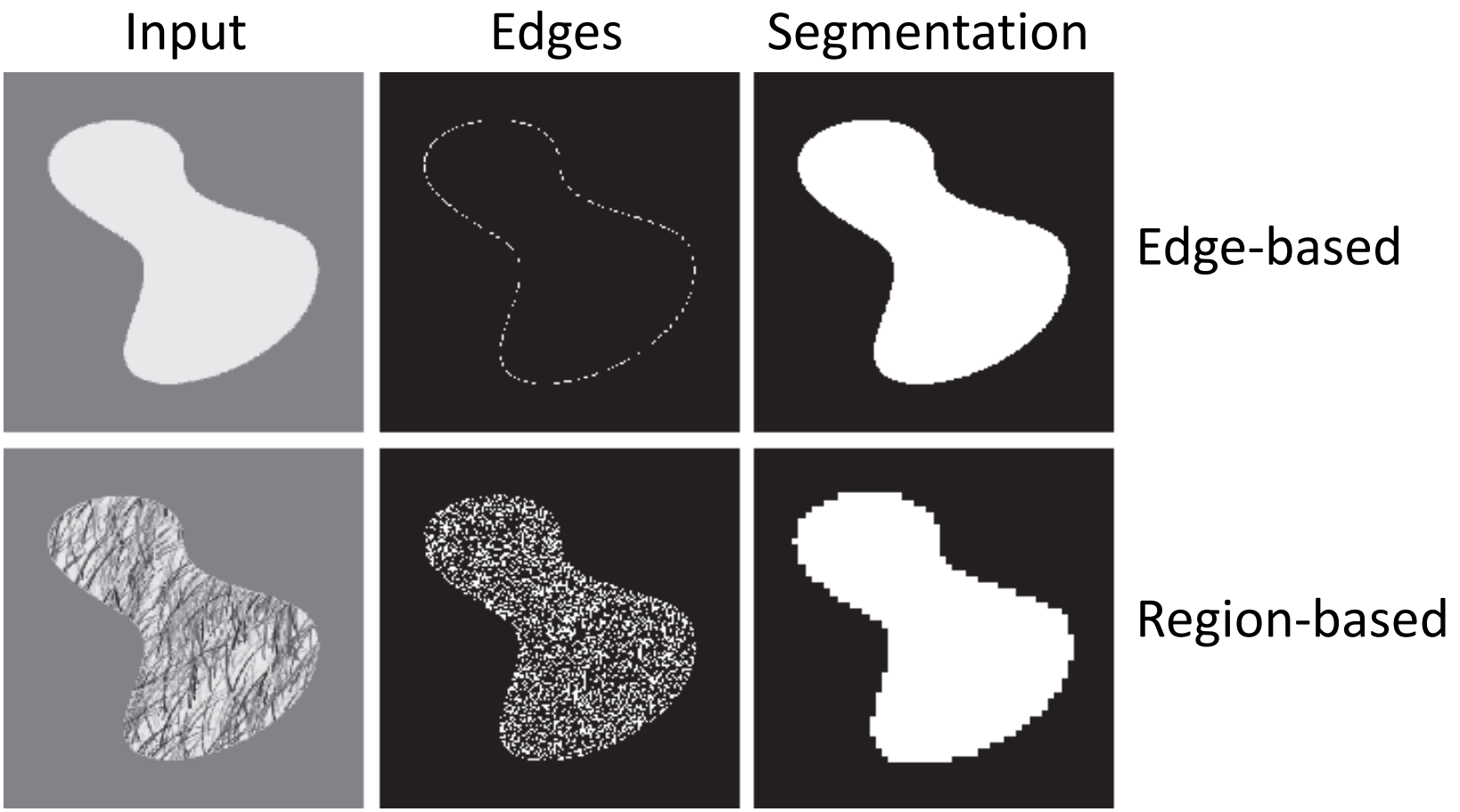
- General approach
  1. Spatial filtering
  2. Additional processing
  3. Thresholding
- Global thresholding (simplest)

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \\ 0 & \text{otherwise} \end{cases}$$

where

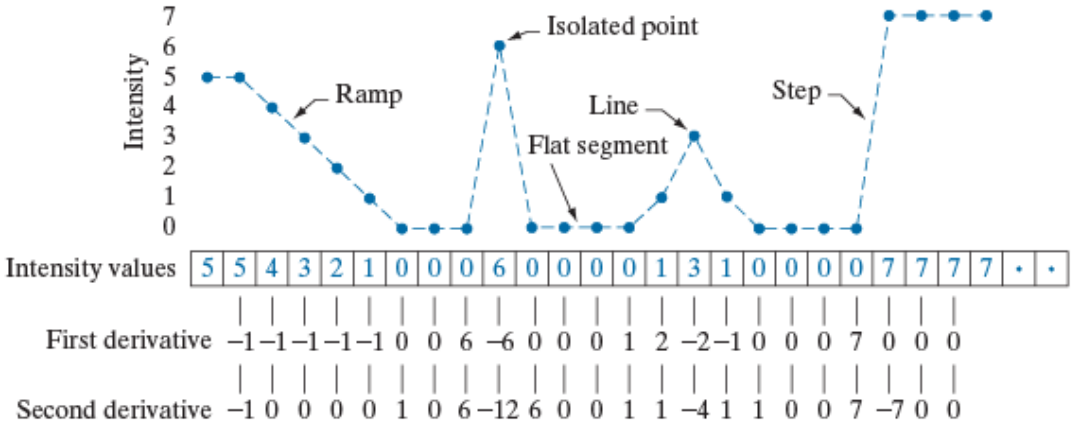
T is threshold value

Image segmentation



Point, line, and edge detection

- Based on image intensity derivatives



See lecture 4

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f(x+1) - f(x) \qquad \text{First derivative (forward difference)}$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = f(x+1) + f(x-1) - 2f(x) \qquad \text{Second derivative}$$

Derivatives revisited

The derivative of a function $f(x)$ can be given by

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \left(= \frac{\Delta y}{\Delta x}, \text{ where } y = f(x) \right)$$

To approximate the derivative at x of $f(x)$, Taylor series expansion of $f(x+\Delta x)$

$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \cdots$$

then use linear terms and solve for $\partial f(x)/\partial x$

$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x}$$

$$f(x+\Delta x) - f(x) = \Delta x \frac{\partial f(x)}{\partial x}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\partial f(x)}{\partial x} \quad \text{forward difference}$$

Derivatives revisited

Similarly, the derivative of a function $f(x)$ can be given by

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} \left(= \frac{\Delta y}{\Delta x}, \text{ where } y = f(x) \right)$$

To approximate the derivative at x of $f(x)$, Taylor series expansion of $f(x - \Delta x)$

$$f(x - \Delta x) = f(x) - \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} - \cdots$$

then use linear terms and solve for $\partial f(x)/\partial x$

$$f(x - \Delta x) = f(x) - \Delta x \frac{\partial f(x)}{\partial x}$$

$$f(x - \Delta x) - f(x) = -\Delta x \frac{\partial f(x)}{\partial x}$$

$$\frac{f(x) - f(x - \Delta x)}{\Delta x} = \frac{\partial f(x)}{\partial x} \quad \text{backward difference}$$

Derivatives revisited

To be symmetric about x, use linear terms of both and solve for $\partial f(x)/\partial x$

$$f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x} \qquad f(x-\Delta x) = f(x) - \Delta x \frac{\partial f(x)}{\partial x}$$

$$f(x+\Delta x) - \Delta x \frac{\partial f(x)}{\partial x} = f(x) \qquad f(x-\Delta x) + \Delta x \frac{\partial f(x)}{\partial x} = f(x)$$

$$f(x+\Delta x) - \Delta x \frac{\partial f(x)}{\partial x} = f(x-\Delta x) + \Delta x \frac{\partial f(x)}{\partial x}$$

$$f(x+\Delta x) - f(x-\Delta x) = 2\Delta x \frac{\partial f(x)}{\partial x}$$

$$\frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \frac{\partial f(x)}{\partial x} \quad \text{central difference}$$

which has lower approximation error than the forward or backward difference

Derivatives in 1D

- Forward difference

$$\frac{\partial f(x)}{\partial x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- Backward difference

$$\frac{\partial f(x)}{\partial x} = \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

- Central difference

$$\frac{\partial f(x)}{\partial x} = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

Image derivatives

Δx is the separation between samples of $f(x)$. For images, the separation is measured in units of pixels, so $\Delta x = 1$

$$\frac{\partial f(x)}{\partial x} = \frac{f(x+1) - f(x-1)}{2} \quad \text{first derivative}$$

$$\frac{1}{2}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad x\text{-derivative correlation filter coefficients}$$

$$\frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad y\text{-derivative correlation filter coefficients}$$

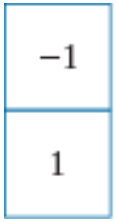
Similarity

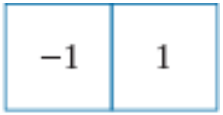
$$\frac{\partial^2 f(x)}{\partial x^2} = f(x+1) - 2f(x) + f(x-1) \quad \text{second derivative}$$

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad x \text{ second derivative filter coefficients}$$

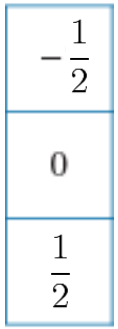
$$\begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \quad y \text{ second derivative filter coefficients}$$

Gradient operators



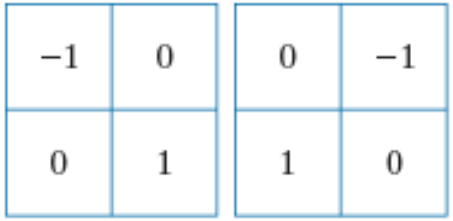


Forward difference

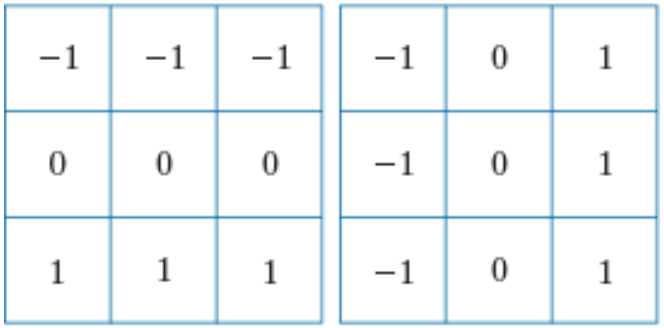




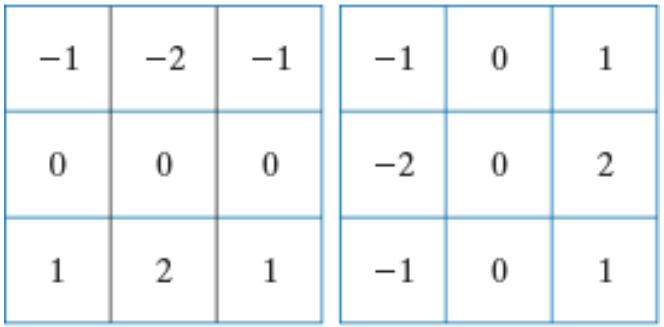
Central difference



Roberts

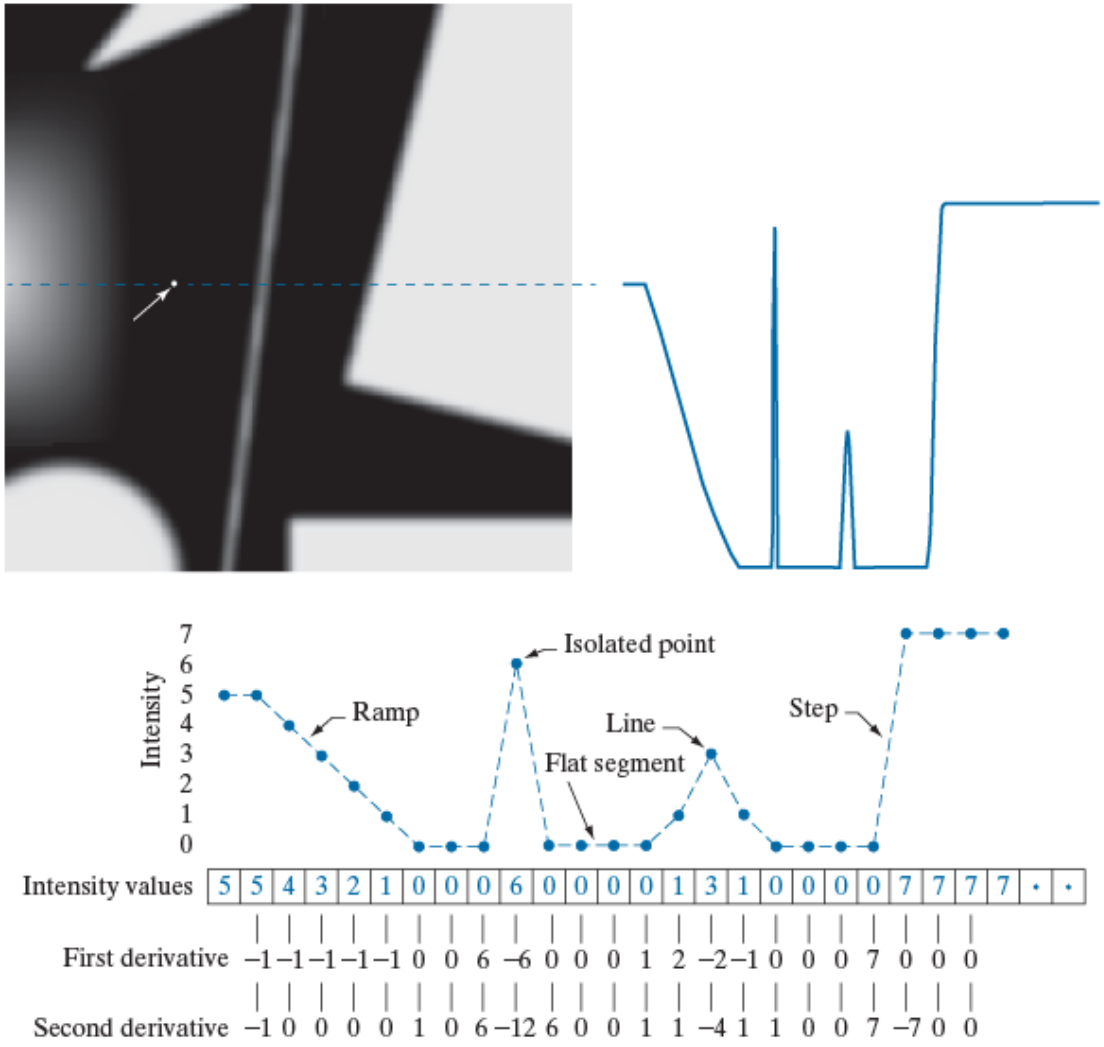


Prewitt

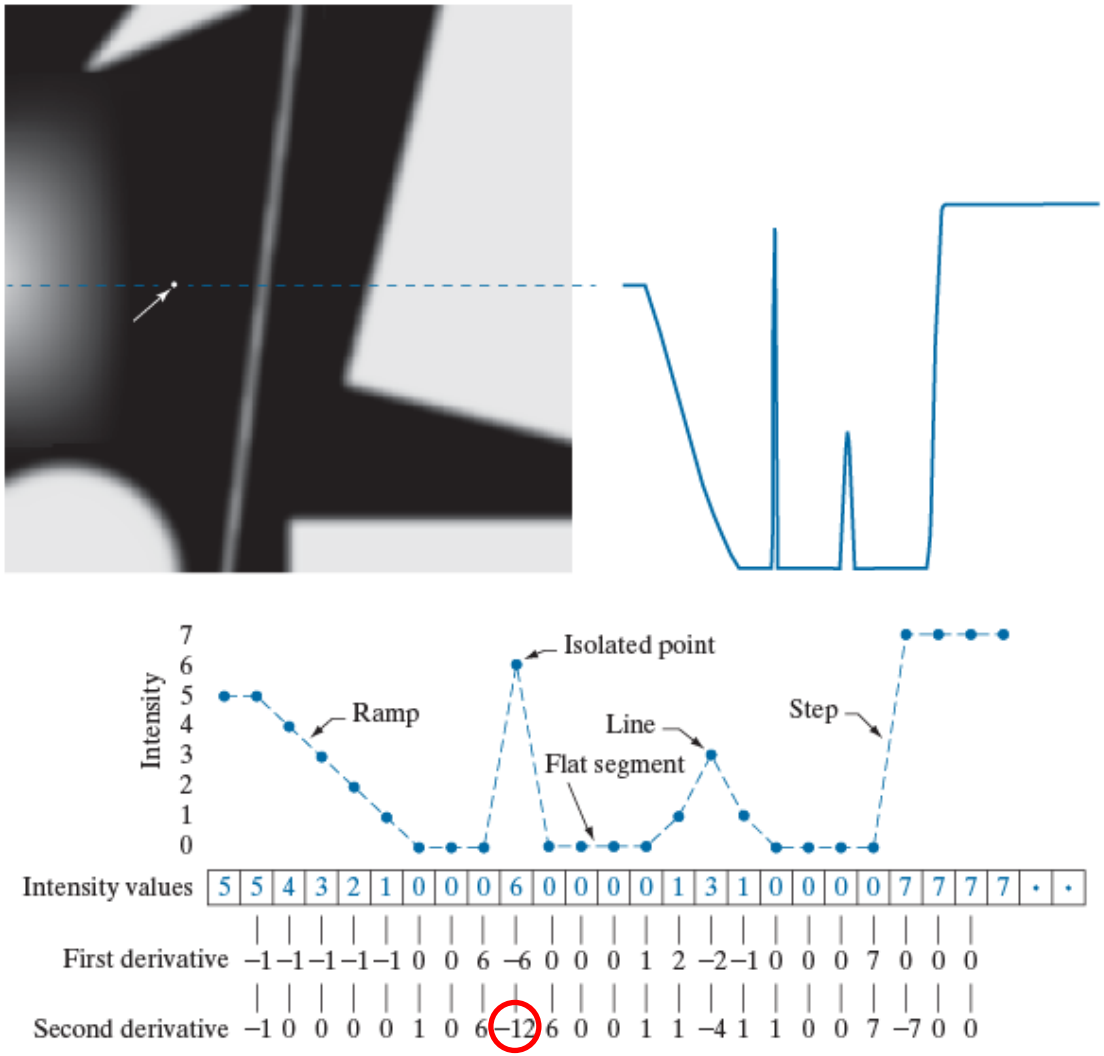


Sobel

Image derivatives



Detection of isolated points



Detection of isolated points

Use second derivative

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \quad \text{Laplacian}$$

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{Laplacian filter coefficients}$$

If diagonal derivatives are also included, then

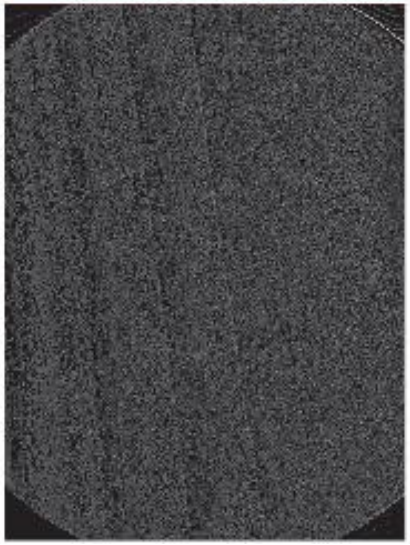
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{Laplacian filter coefficients}$$

Detection of isolated points

1	1	1
1	−8	1
1	1	1

Laplacian
(second derivative)







Threshold
absolute
value

Input

Segmentation

Line detection

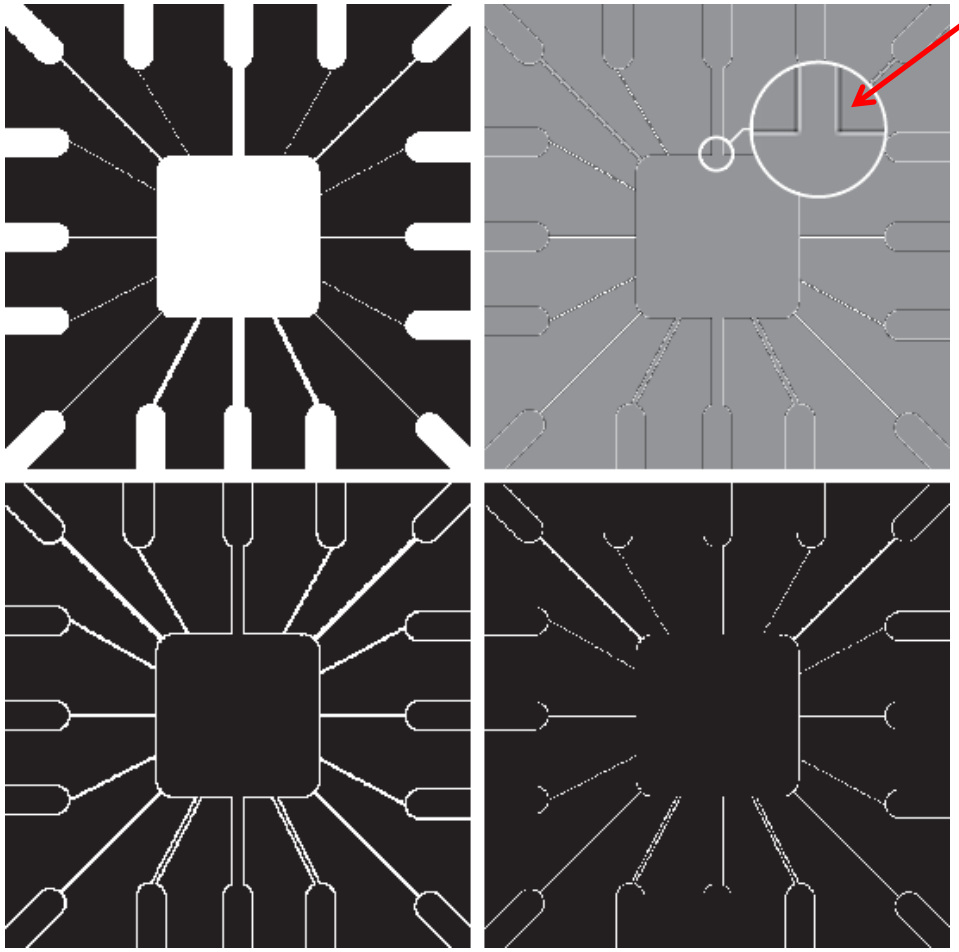
- Also use Laplacian kernel, which is independent of line direction

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ Laplacian filter coefficients

If diagonal derivatives are also included, then

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ Laplacian filter coefficients

Line detection



Line detection, specific directions

- For specific directions, use directional kernels

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

2	−1	−1
−1	2	−1
−1	−1	2

+45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

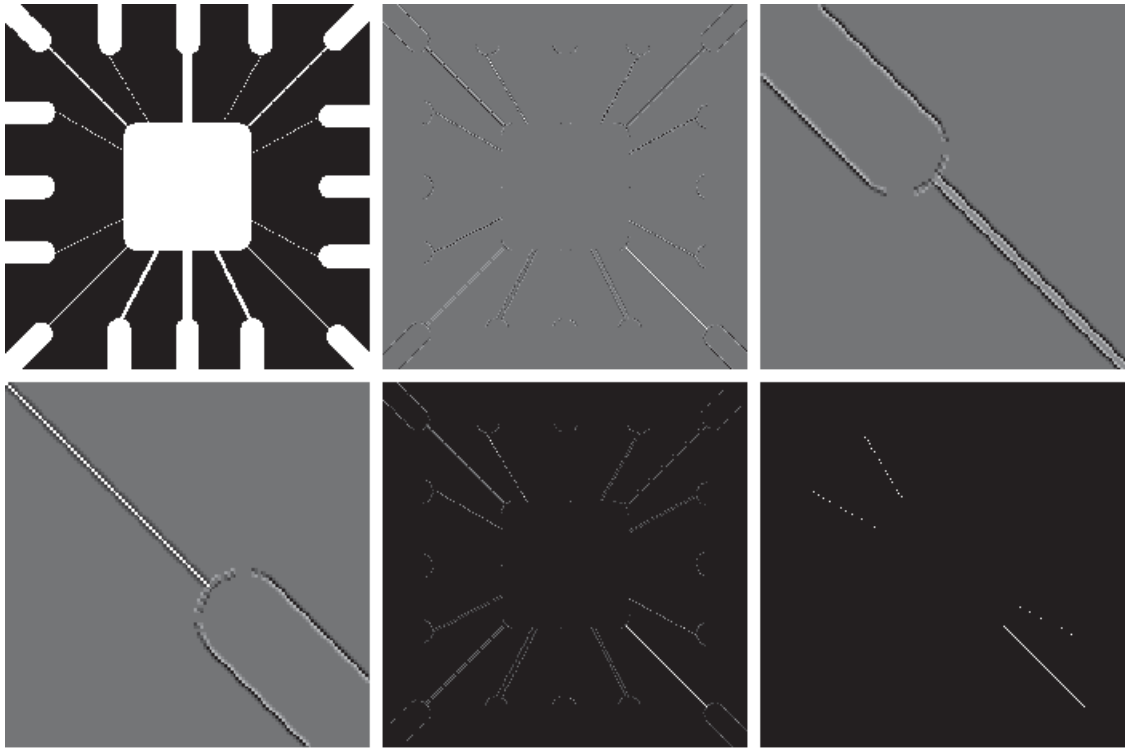
−1	−1	2
−1	2	−1
2	−1	−1

−45°

Spatial filters

Line detection, specific directions

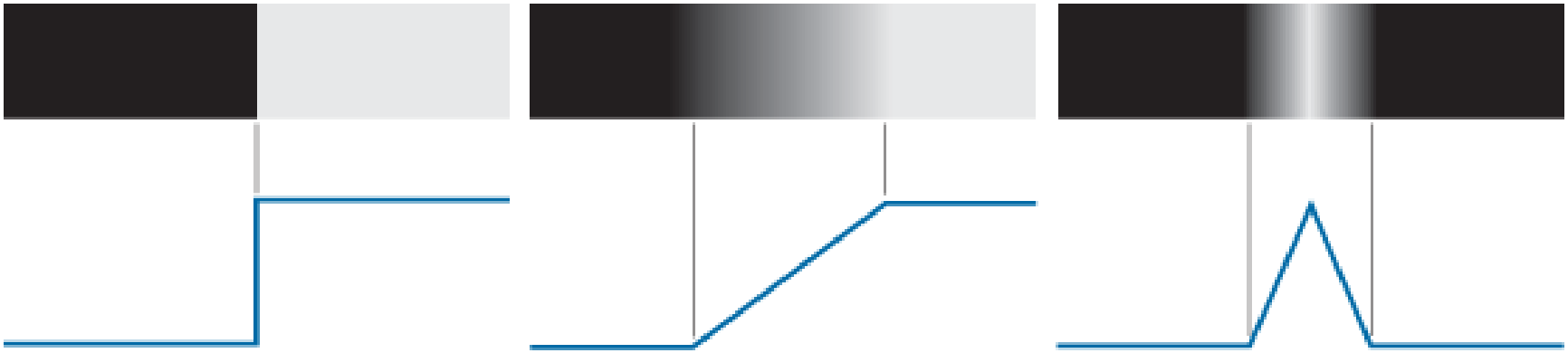
+45°



Negative values set to zero

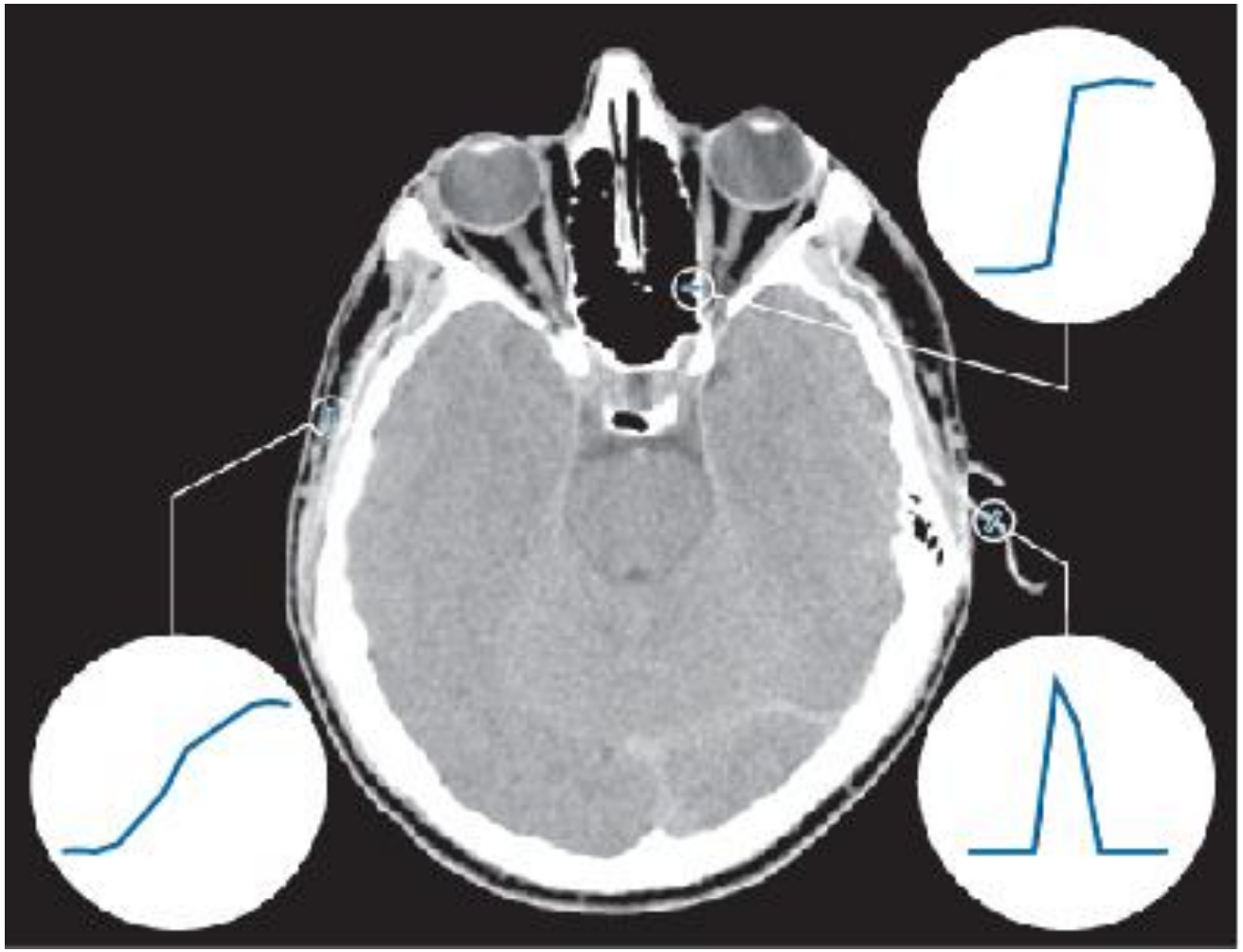
Threshold

Edge models



Step Ramp Roof edge

Edge models

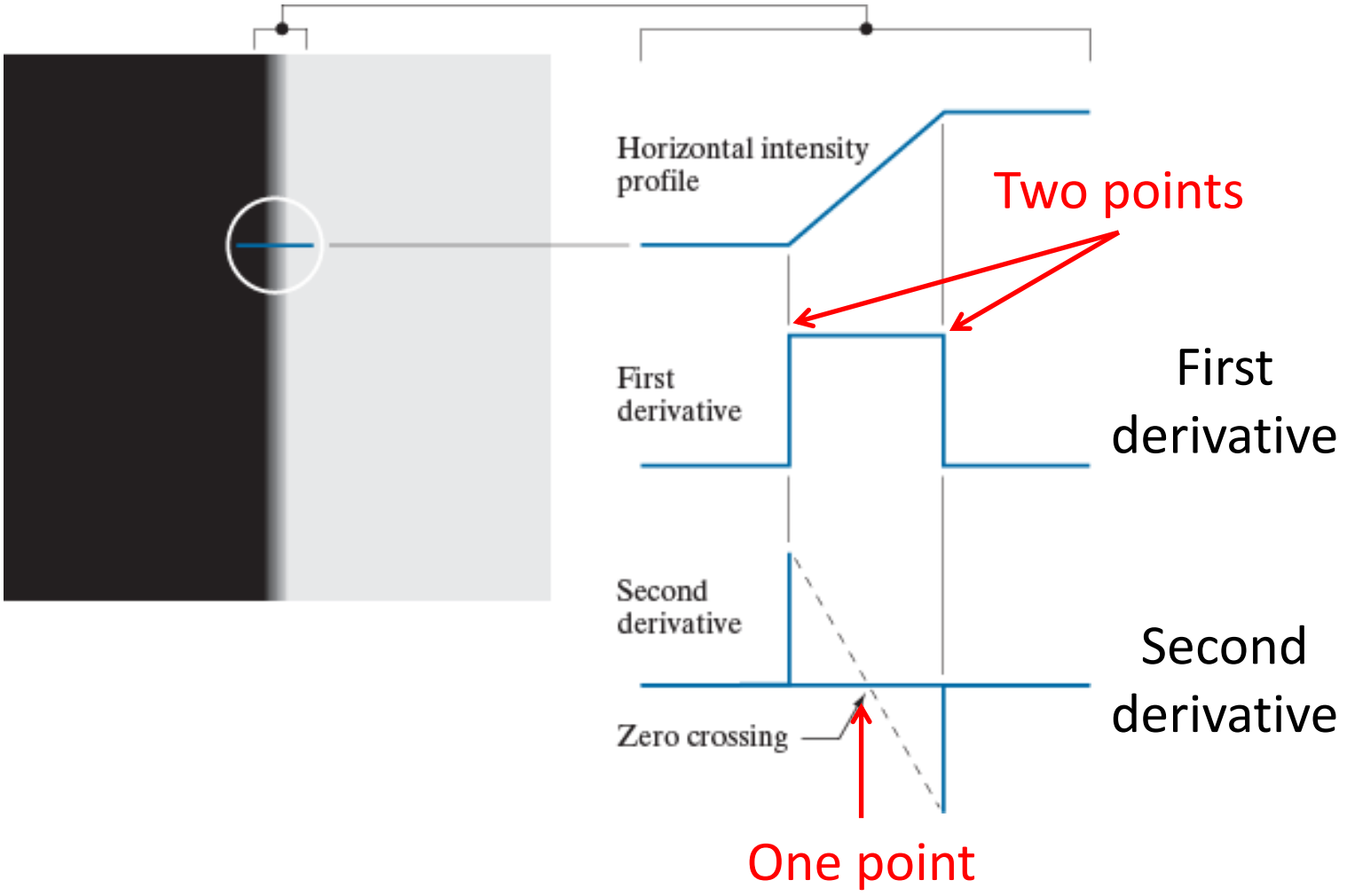


Step

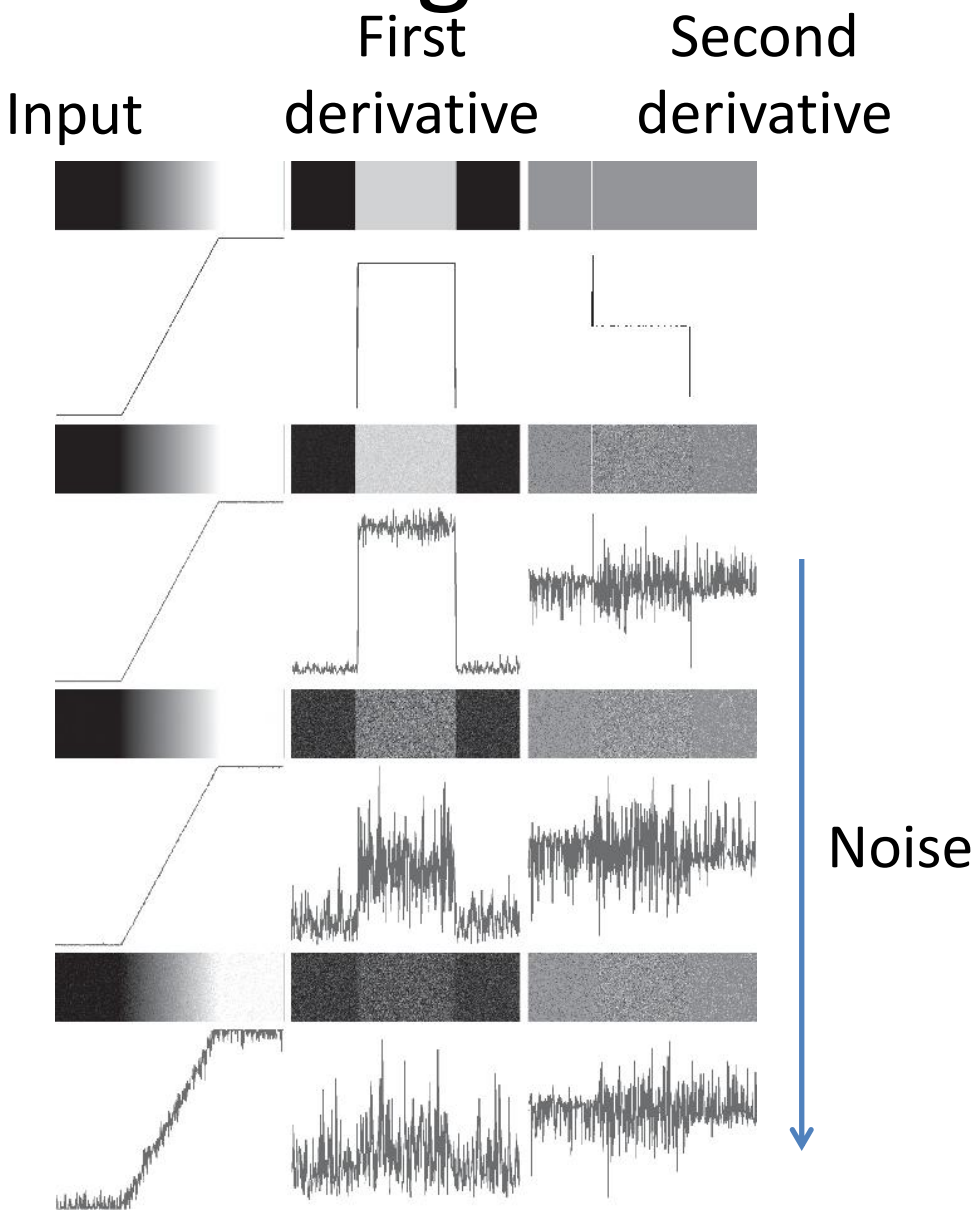
Ramp

Roof edge

Ramp edge


Horizontal intensity profile
Two points
First derivative
First derivative
Second derivative
Second derivative
Zero crossing
One point

Noise and image derivatives



Edge detection

- Clearly, noise must be reduced
- Approach (simple)
  1. Image smoothing for noise reduction
  2. Detection of image points (edge point candidates)
  3. Edge localization (select from candidates, set of edge points)

Image gradient

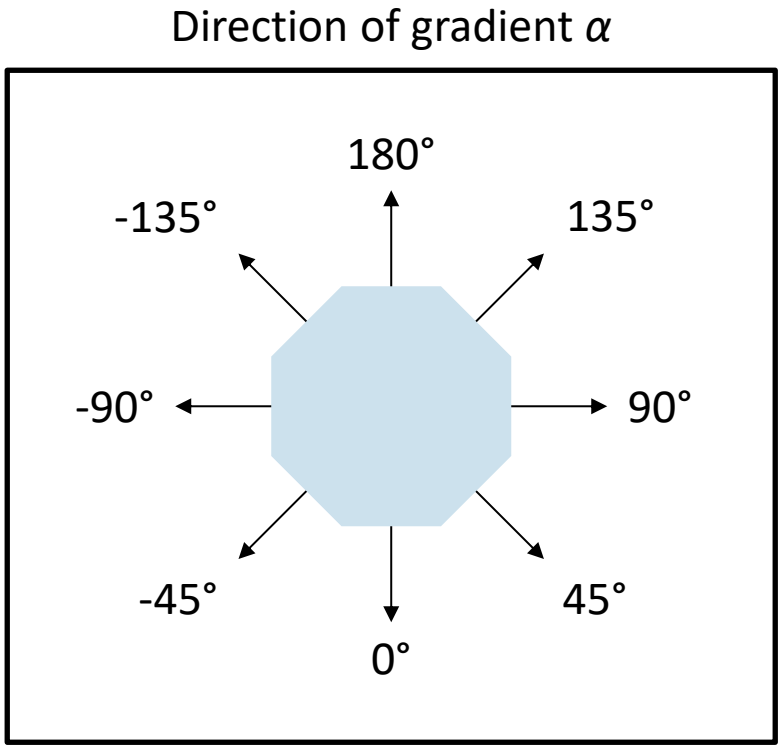
$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} g_x(x,y) \\ g_y(x,y) \end{bmatrix} \quad \text{gradient vector}$$

$$\|\nabla f(x,y)\| = \sqrt{g_x^2(x,y) + g_y^2(x,y)} \quad \text{magnitude of gradient}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{g_y(x,y)}{g_x(x,y)}\right) \quad \text{direction of gradient}$$

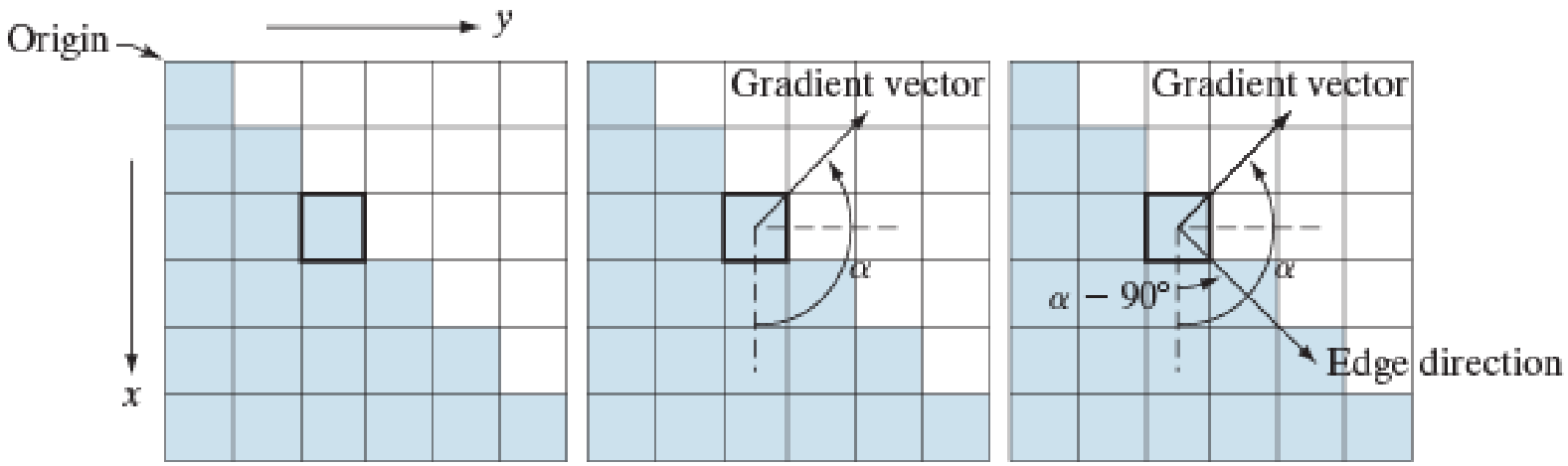
Use atan2

Gradient vector direction



The gradient vector is one of the edge normal vectors.
The other normal edge vector is in the opposite direction of the gradient vector.

Gradient and edge direction

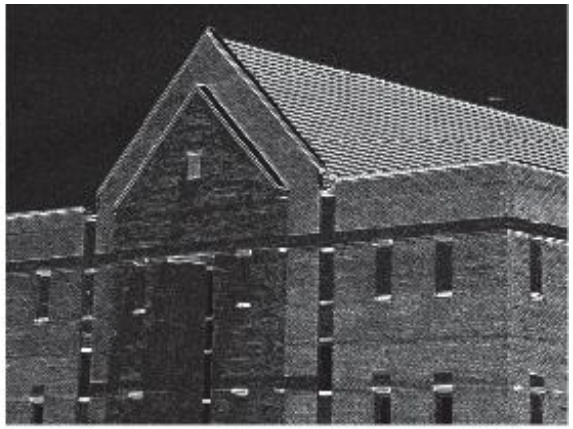


Gradient direction is orthogonal to edge direction

Gradients

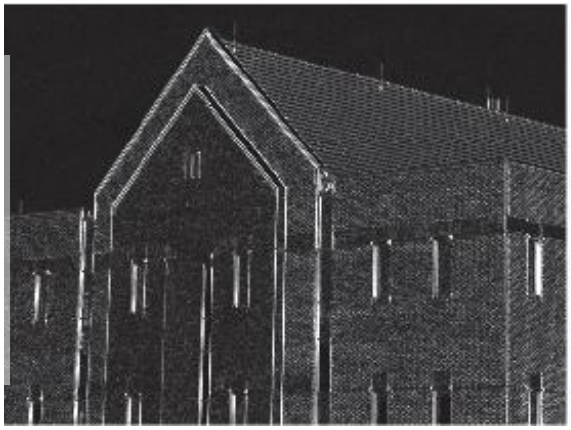
Input

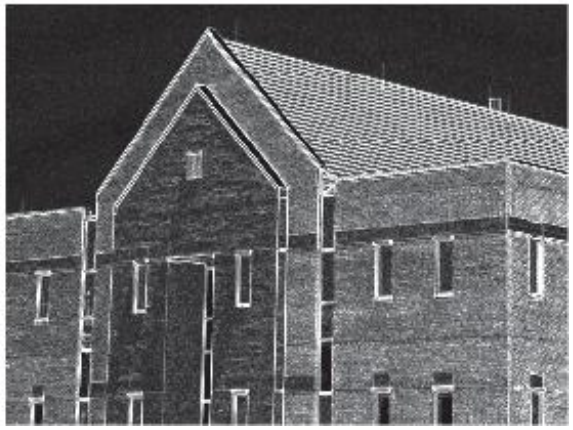




Magnitude of vertical gradient

Magnitude of horizontal gradient





Magnitude of gradient vector

Gradients

Smooth image prior to computing gradients.
Results in more selective edges

Input





Magnitude of vertical gradient

Magnitude of horizontal gradient





Magnitude of gradient vector

Edge detection

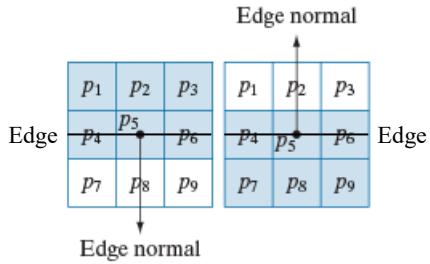
1. Smooth the input image
2. Compute the gradient magnitude and gradient direction images
3. Apply nonmaximal suppression to the gradient magnitude image using the gradient direction image
4. Threshold the resulting image

Nonmaxima suppression along the gradient direction

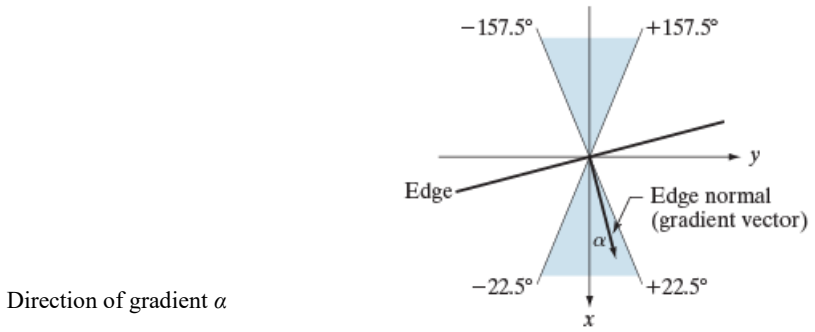
Specify a number of discrete orientations $d_1, d_2, \ldots$

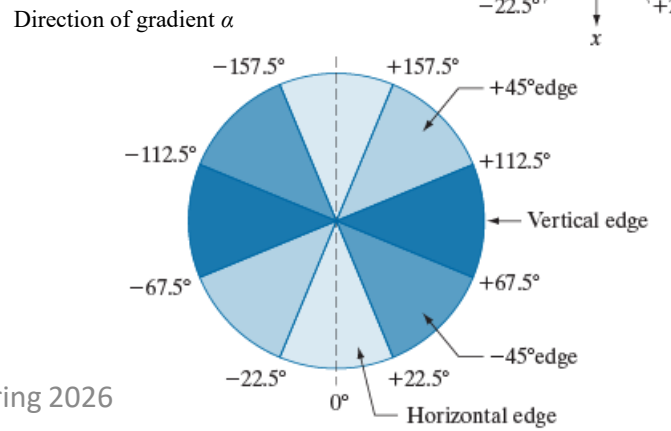
1. Determine the direction d_k closest to $\alpha(x, y)$
2. Let K denote the value of $\|\nabla f\|$ at (x, y). If K is less than the value of $\|\nabla f\|$ at one or both of the neighbors of point (x, y) along d_k, let $g_N(x, y) = 0$ (suppression); otherwise, let $g_N(x, y) = K$.

Both of the neighbors means the two pixels corresponding to both of the edge normal vectors



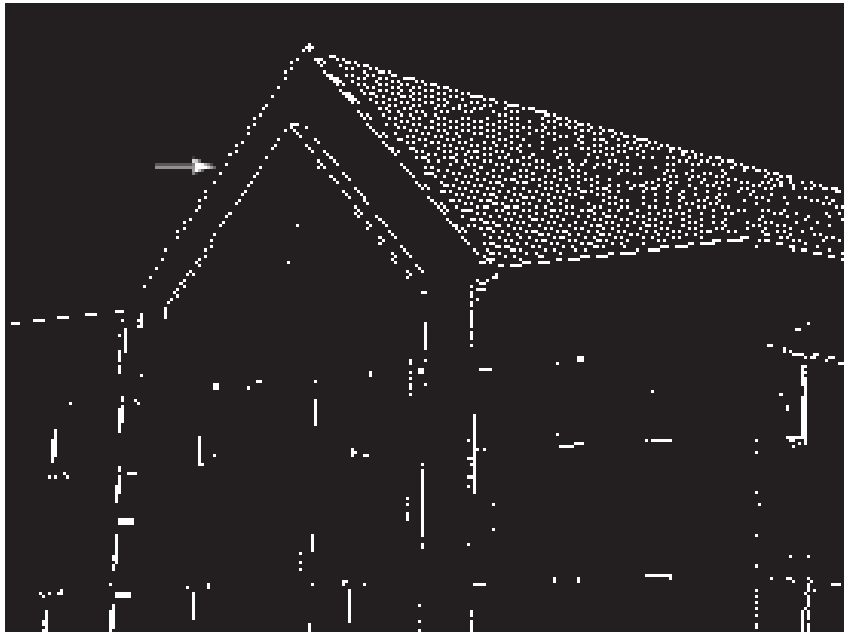
Example: horizontal edge



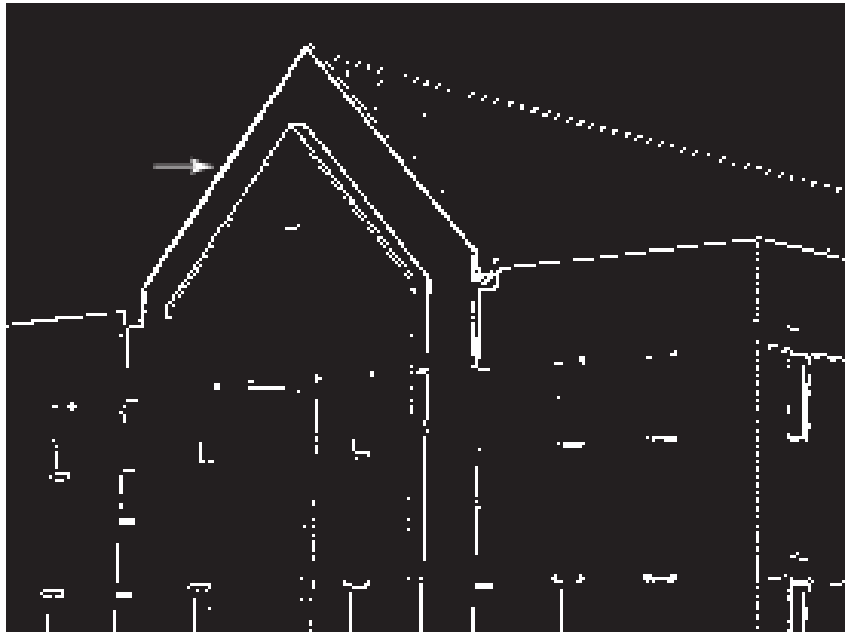


Edge detection

Threshold magnitude of gradient vector



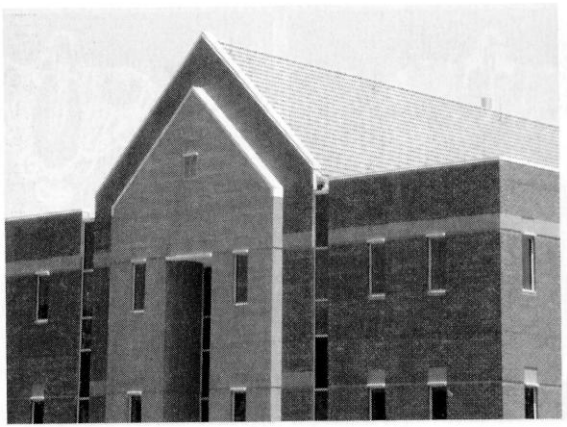
Without smoothing

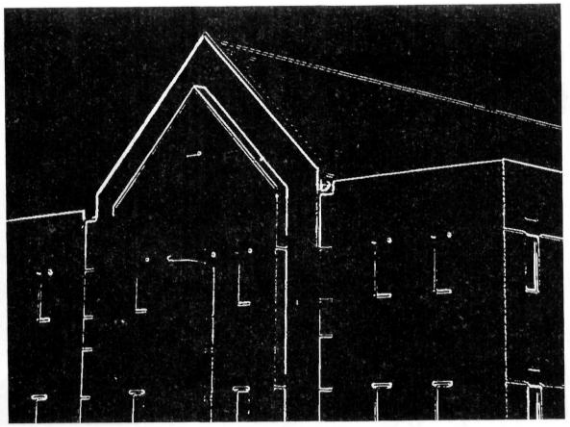


With smoothing

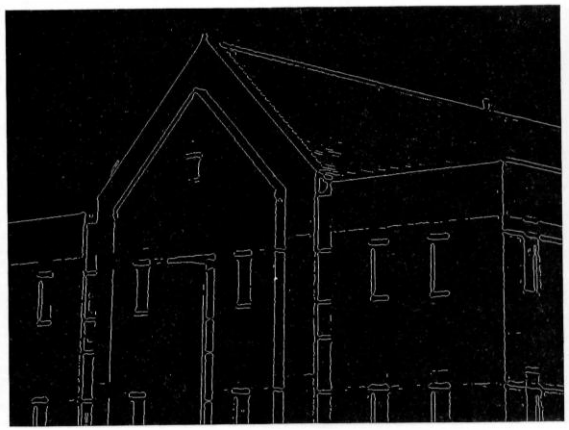
Advanced edge detection

Input

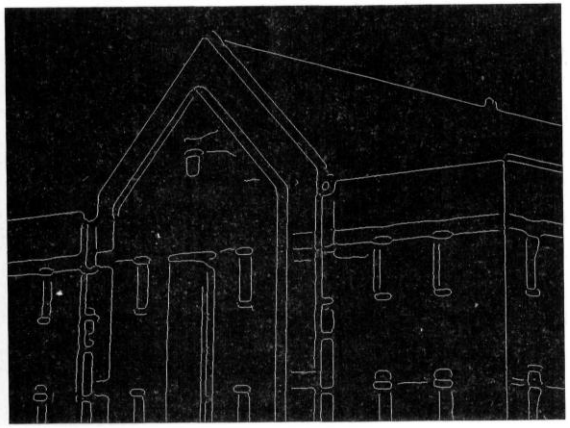




Magnitude of gradient vector (with smoothing)



Marr-Hildreth



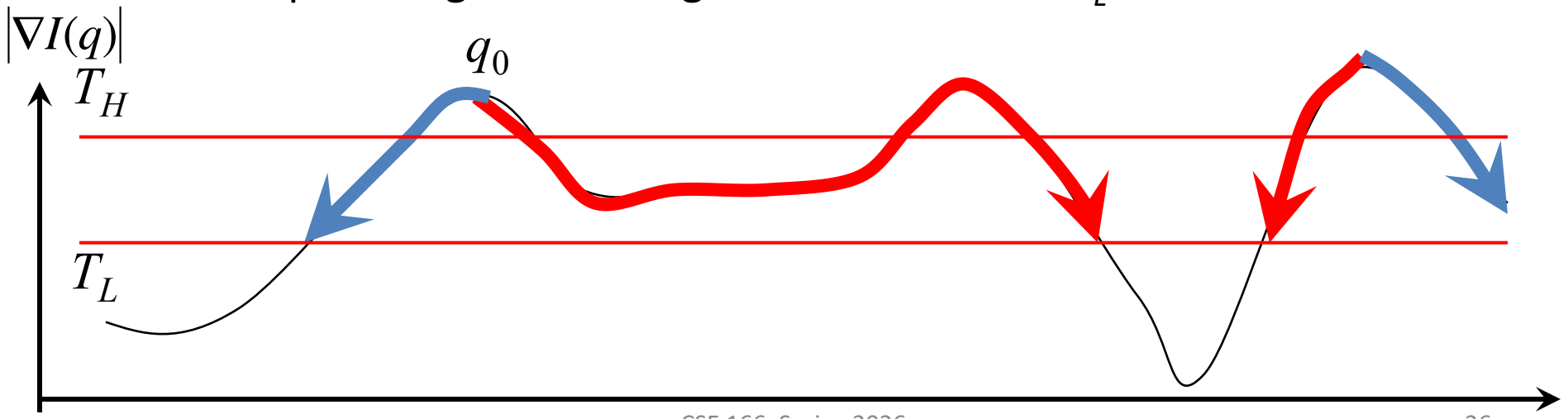
Canny

Canny edge detector

1. Smooth the input image with a Gaussian filter
2. Compute the gradient magnitude and angle images
3. Apply nonmaximal suppression to the gradient magnitude image
4. Use double thresholding and connectivity analysis to detect and link edges

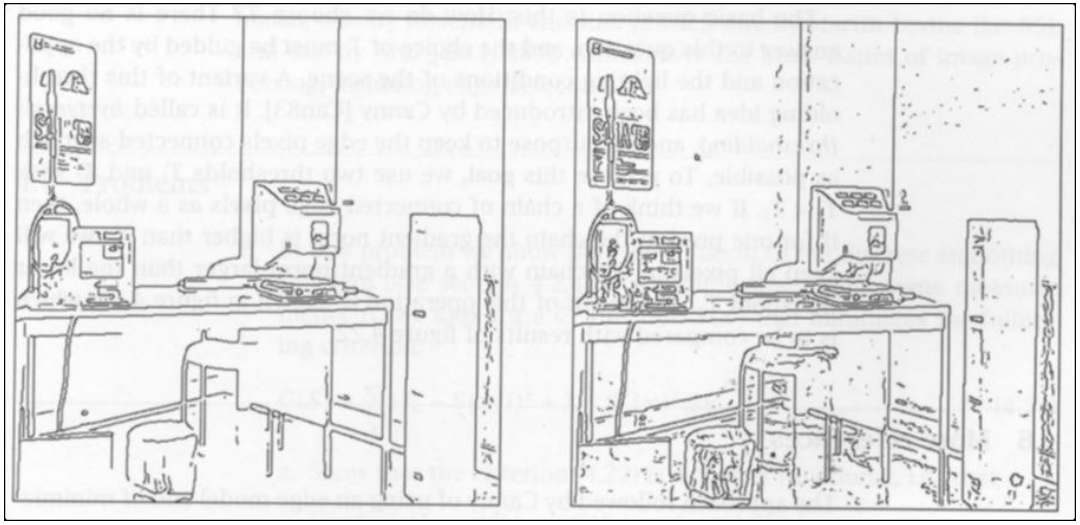
Double thresholding

- Use a high threshold to start edge curves and a low threshold to continue them
  - Define two thresholds T_H and T_L
  - Starting with output of nonmaximal suppression, find a point q_0, which is a local maximum greater than T_H
  - Start tracking an edge chain at pixel location q_0 in one of the two directions
  - Stop when gradient magnitude is less than T_L



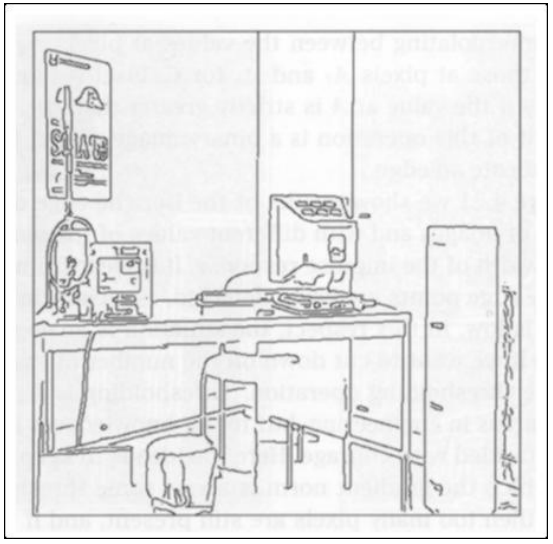
Double thresholding

Single threshold



$T = 15$ $T = 5$

Double threshold



$T_H = 15$ and $T_L = 5$

Canny edge detector



Canny edge detector



Next Lecture

- Image segmentation
- Reading
  - Chapter 10: Image segmentation I: edge detection, thresholding, and region detection
    - Sections 10.3, 10.4, 10.5, and 10.6