Morphological Image Processing

Image Processing

CSE 166

Lecture 13

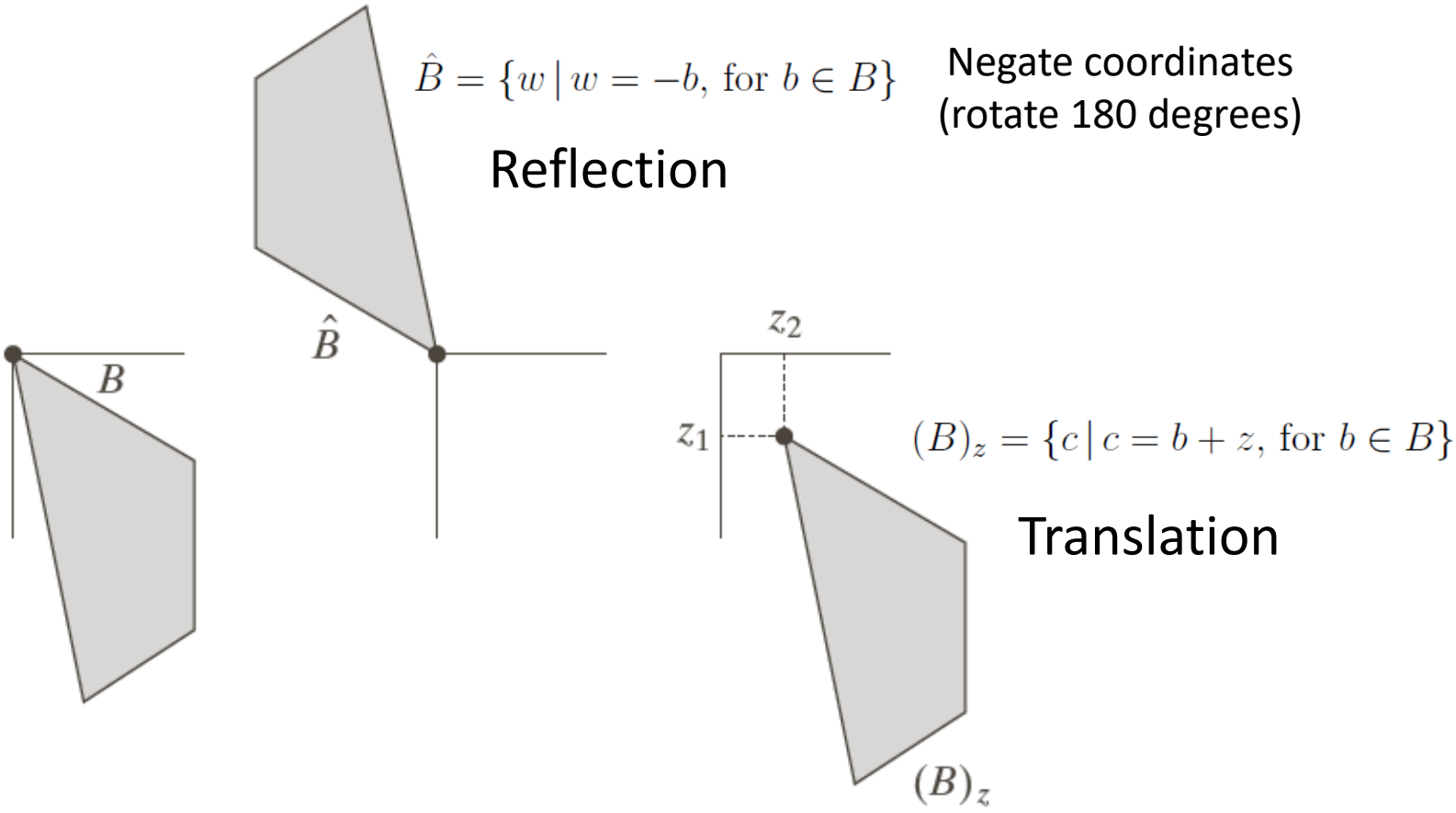
Announcements

- Assignment 5 is due May 20, 11:59 PM
- Assignment 6 will be released May 27
  - Due Jun 3, 11:59 PM
- Reading
  - Chapter 9: Morphological image processing
    - Sections 9.1, 9.2, 9.3, and 9.5 (through subsection connected components)

Morphological image processing

- Mathematical morphology
- Set theory
- Binary image

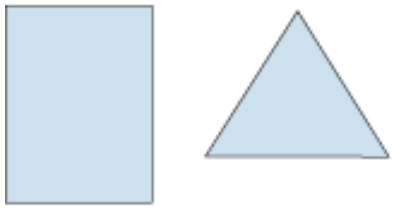
Reflection and translation



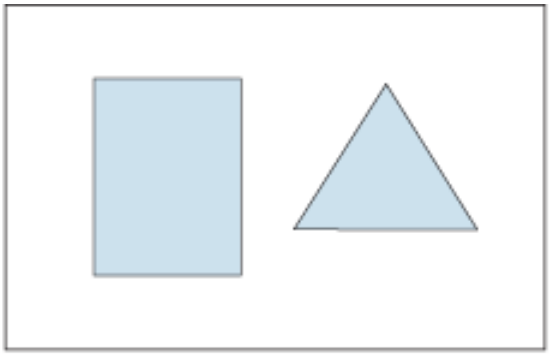
Structuring elements

- Small sets used to probe an image
  - Origin is commonly in the center of the structuring element (similar to spatial filtering kernels)

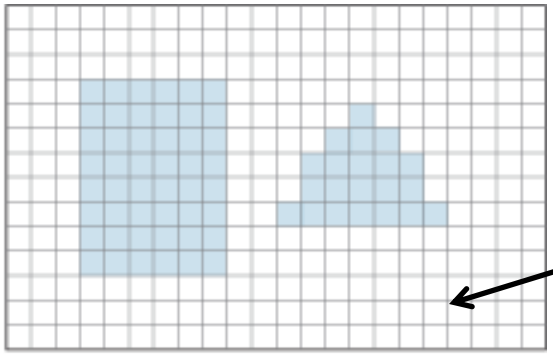
Sets of pixels: objects and structuring elements (SEs)



Objects representeed as sets



Objects represented as a graphical image



Digital image

Border of background pixels around objects



Structuring element represented as a set



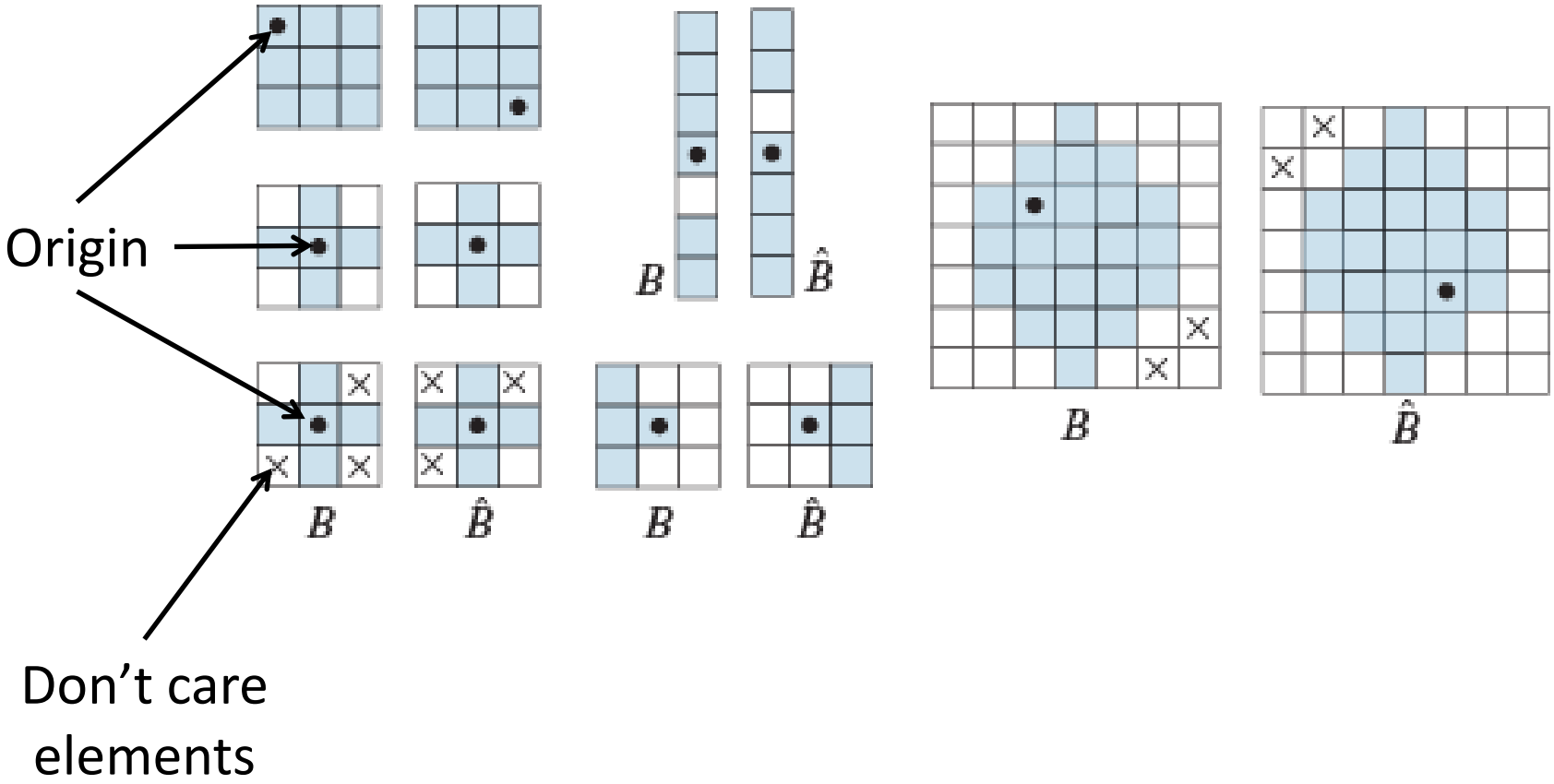
Structuring element represented as a graphical image



Digital structuring element

Tight border around SE

Reflection about the origin



Morphological operations

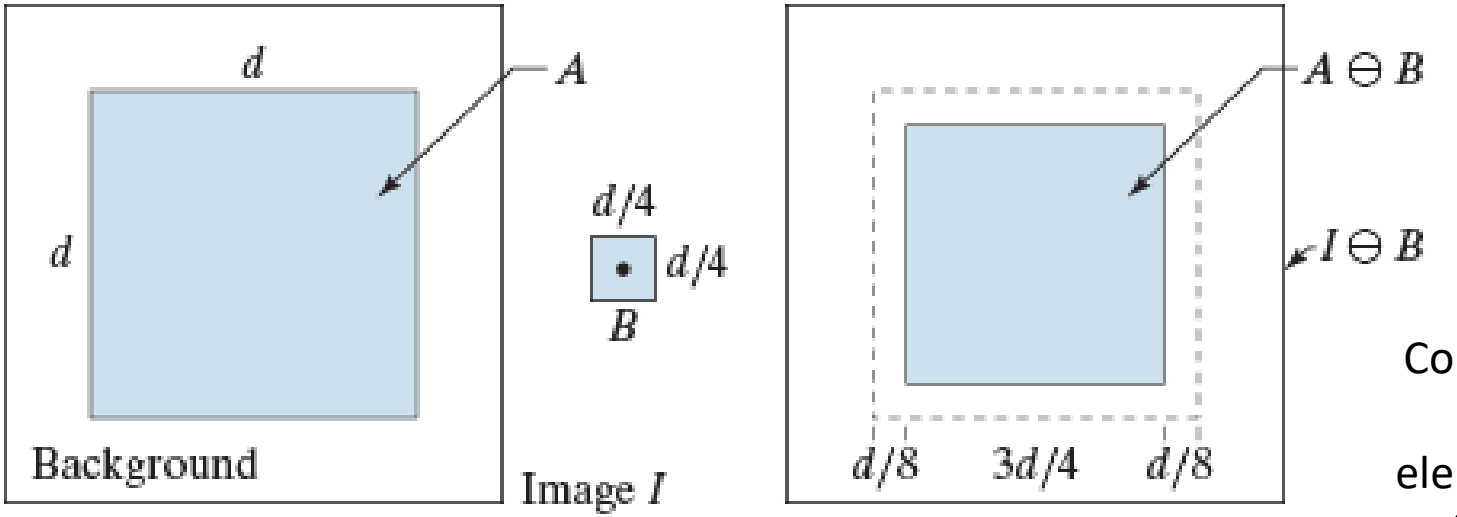
- Erosion
- Dilation
- Opening
- Closing

Erosion

- Example: square SE
  - The erosion of A by B

B translated by z, is contained in A

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



Complement of A (i.e., set of elements not in A)

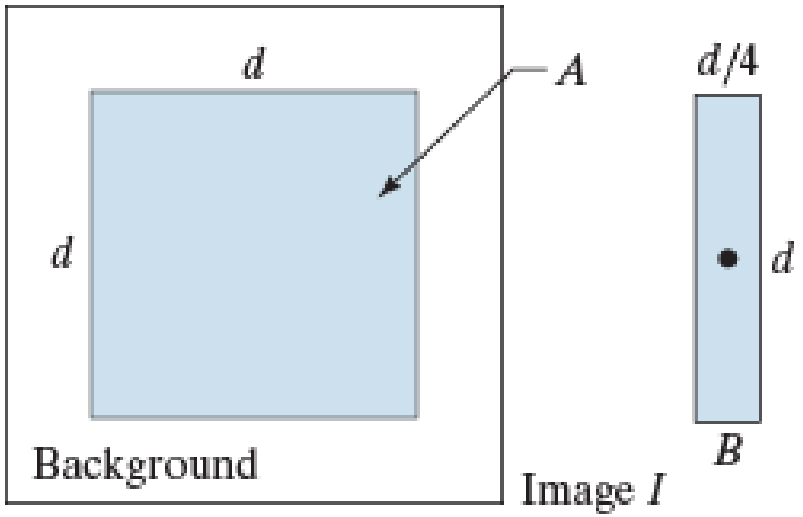
$$I \ominus B = \{z \mid (B)_z \subseteq A \text{ and } A \in I\} \cup \{A^c \mid A^c \subseteq I\}$$

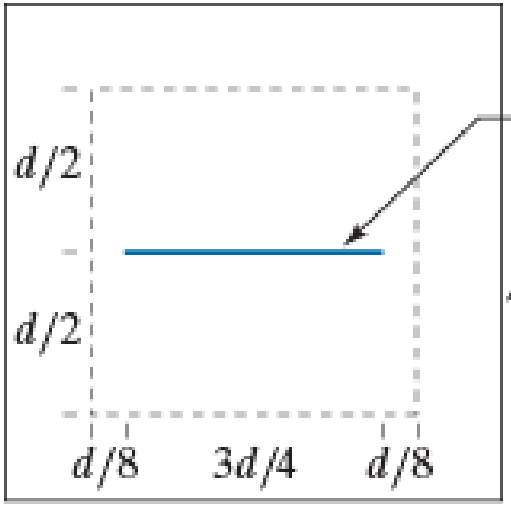
Erosion

- Example: elongated SE
  - The erosion of A by B

B translated by z, is contained in A

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



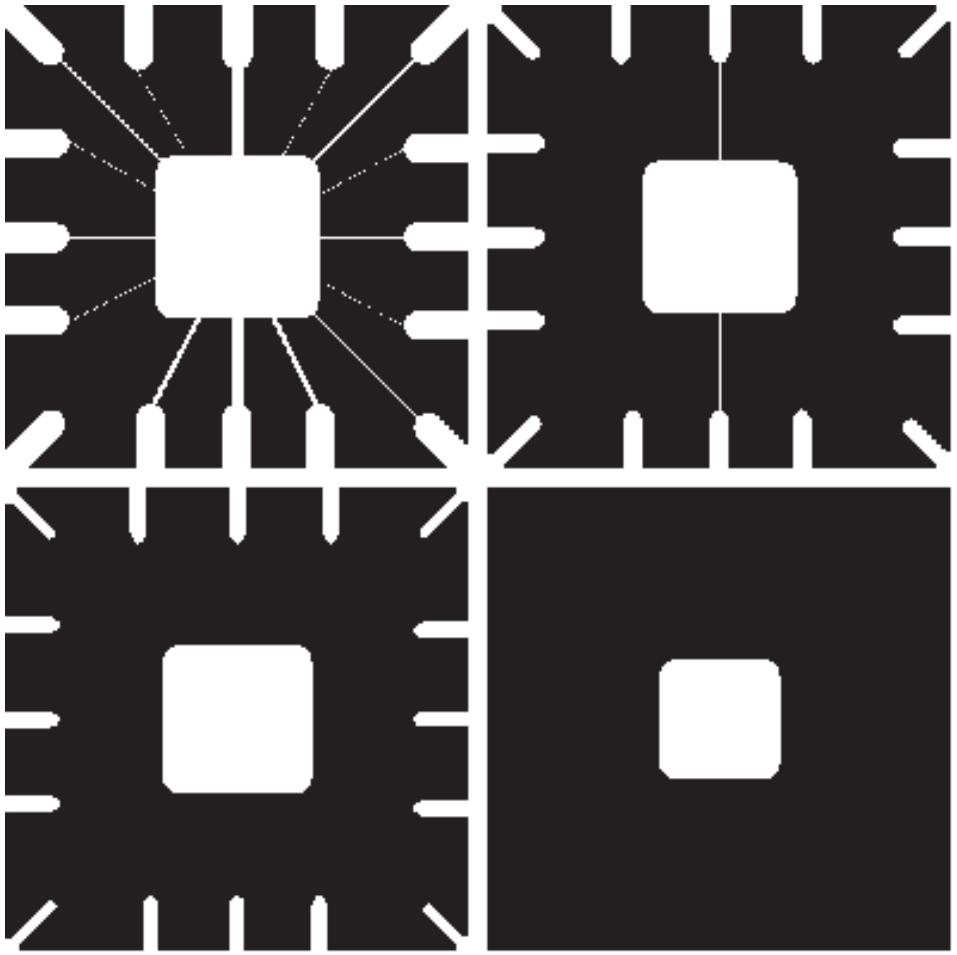


Complement of A (i.e., set of elements not in A)

$$I \ominus B = \{z \mid (B)_z \subseteq A \text{ and } A \in I\} \cup \{A^c \mid A^c \subseteq I\}$$

Erosion

Equivalent to a min filter
Shrinks



11x11

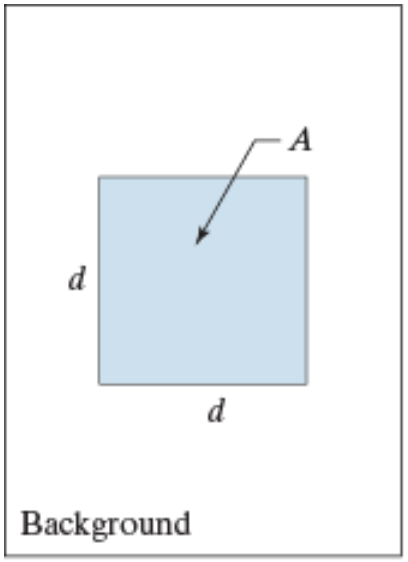
15x15

45x45

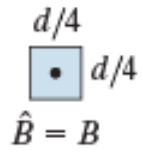
Dilation

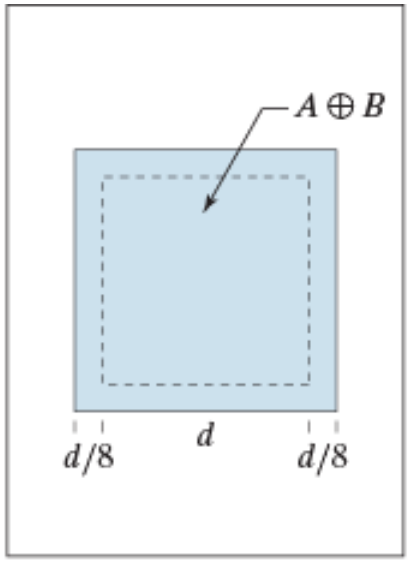
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

Examples



Image, I

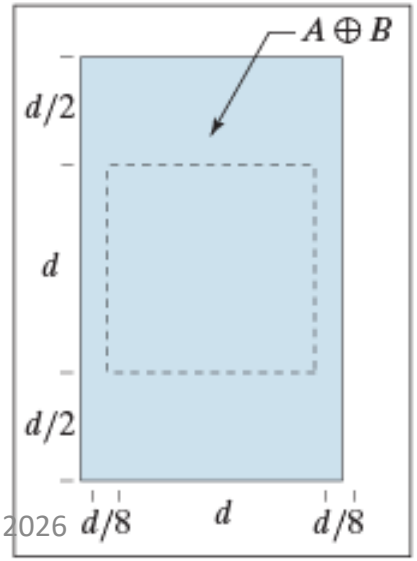




$I \oplus B$

Square SE





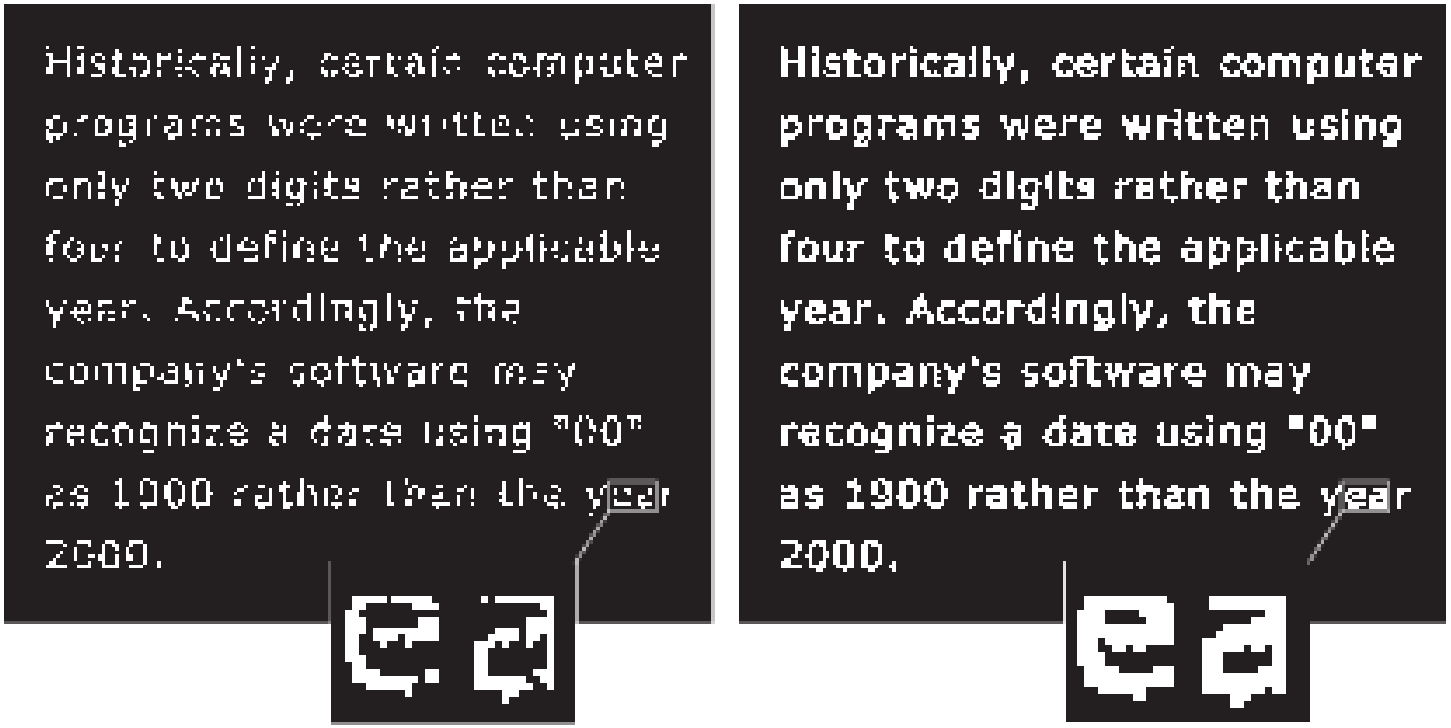
$I \oplus B$

Elongated SE

Dilation

Equivalent to a max filter

Expands



1	1	1
1	1	1
1	1	1

Duality

- Erosion and dilation are duals of each other with respect to complementation and reflection

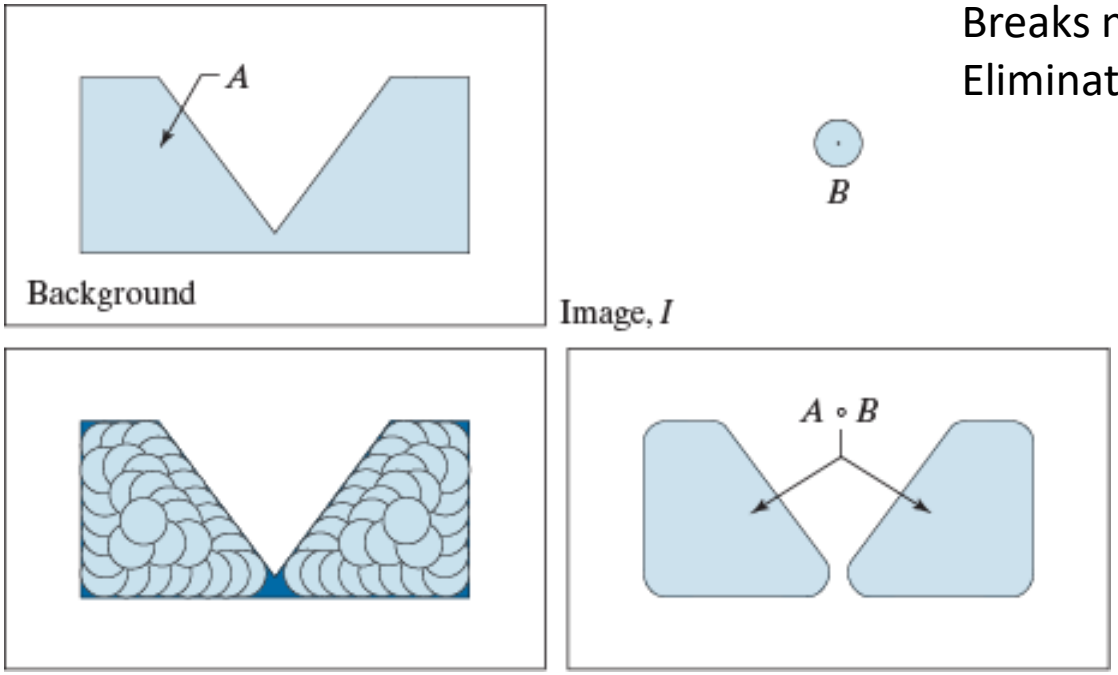
$$(A \ominus B)^c = A^c \oplus \hat{B} \text{ and } (A \oplus B)^c = A^c \ominus \hat{B}$$

Opening

- Opening a set A by structuring element B
  - Erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

Smooths the contour of an object
Breaks narrow isthmuses
Eliminates thin protrusions

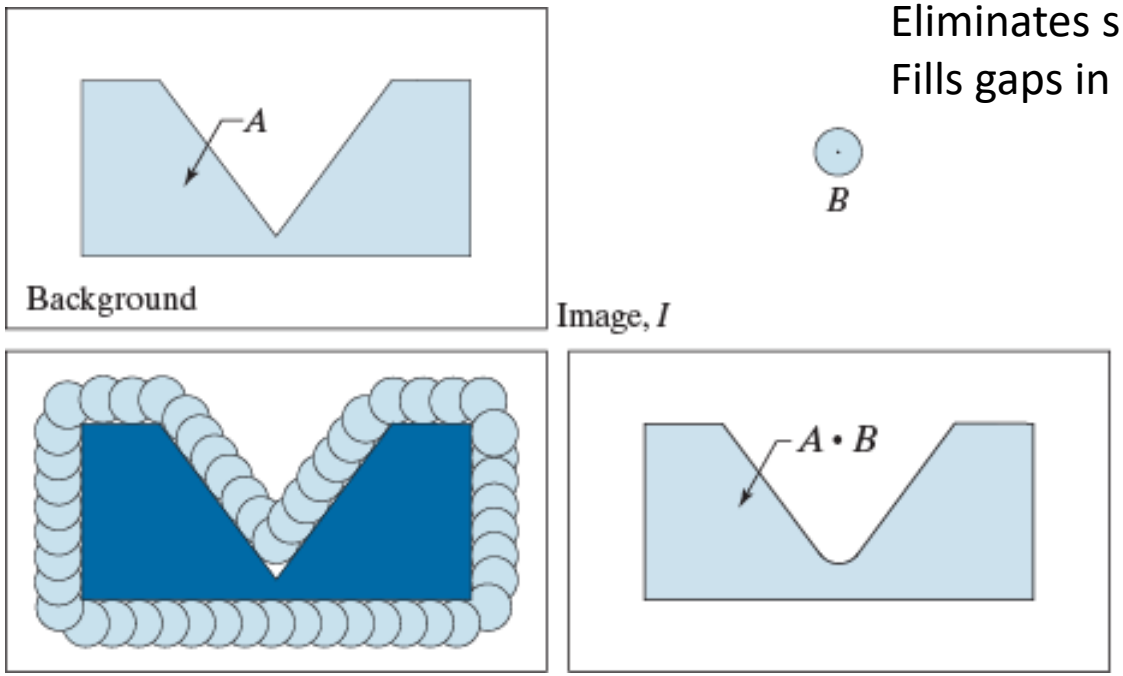


Structuring element rolls along **inner** boundary

Closing

- Closing a set A by structuring element B
  - Dilation followed by erosion $A \bullet B = (A \oplus B) \ominus B$

Fuses narrow breaks and long thin gulfs
Eliminates small holes
Fills gaps in contour of an object

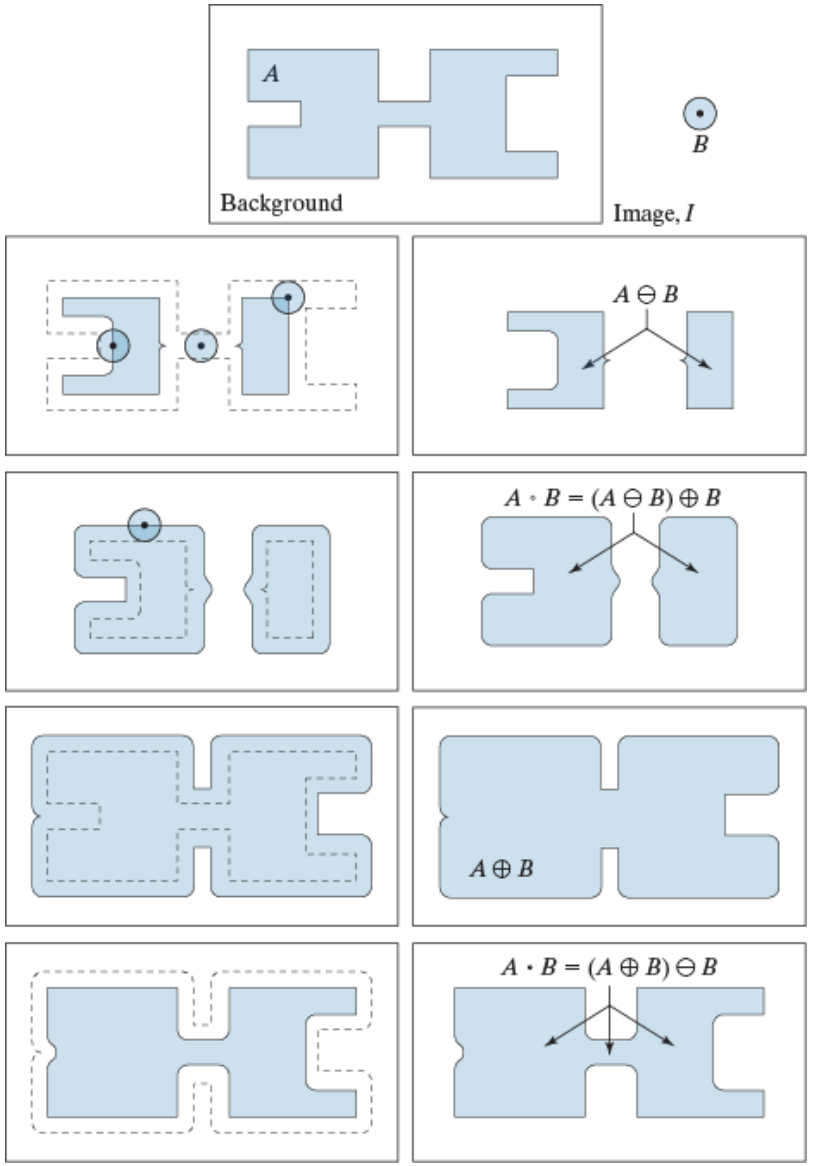


Structuring element rolls along **outer** boundary

Duality

- Opening and closing are duals of each other with respect to complementation and reflection

Basic morphological operations



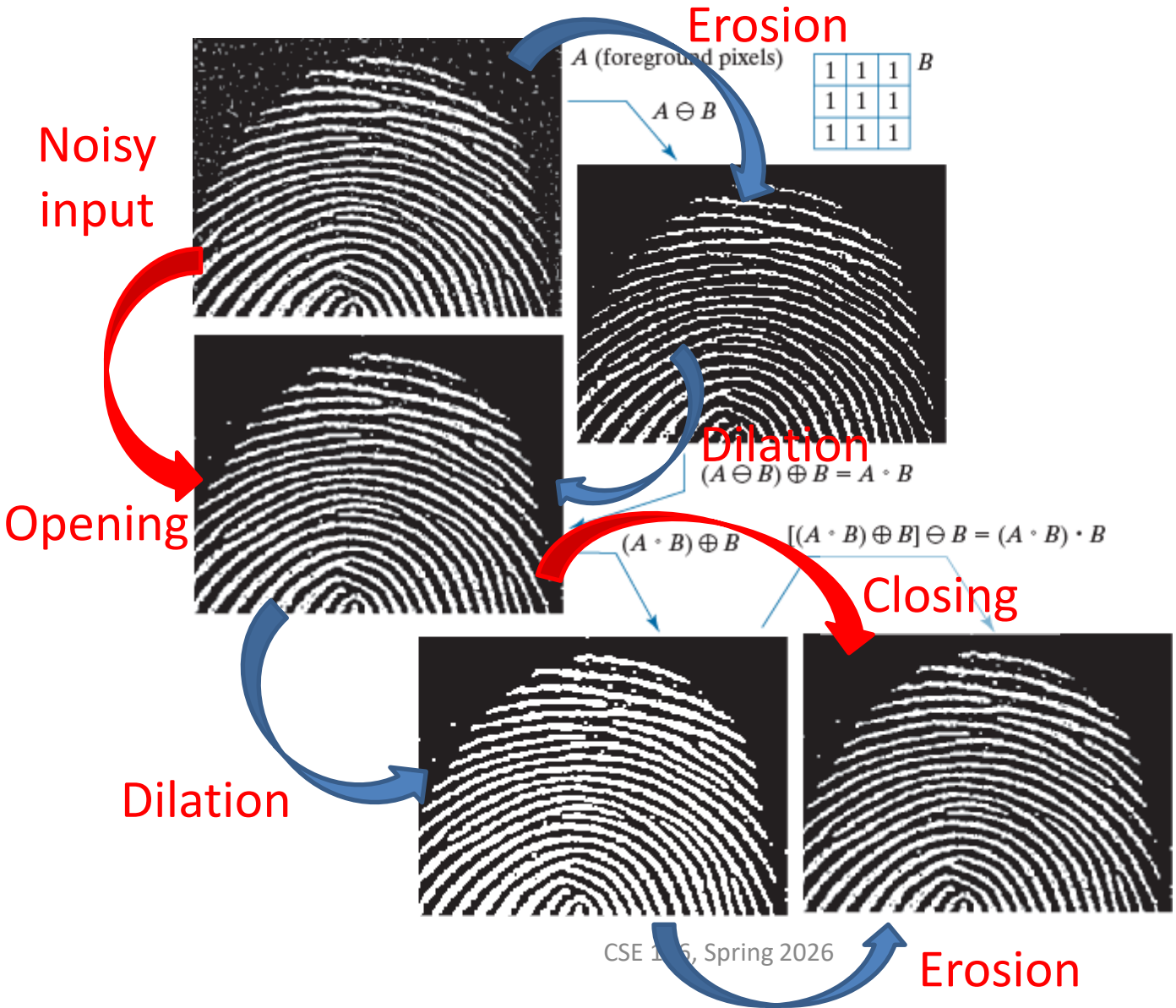
Erosion

Opening

Dilation

Closing

Morphological image processing



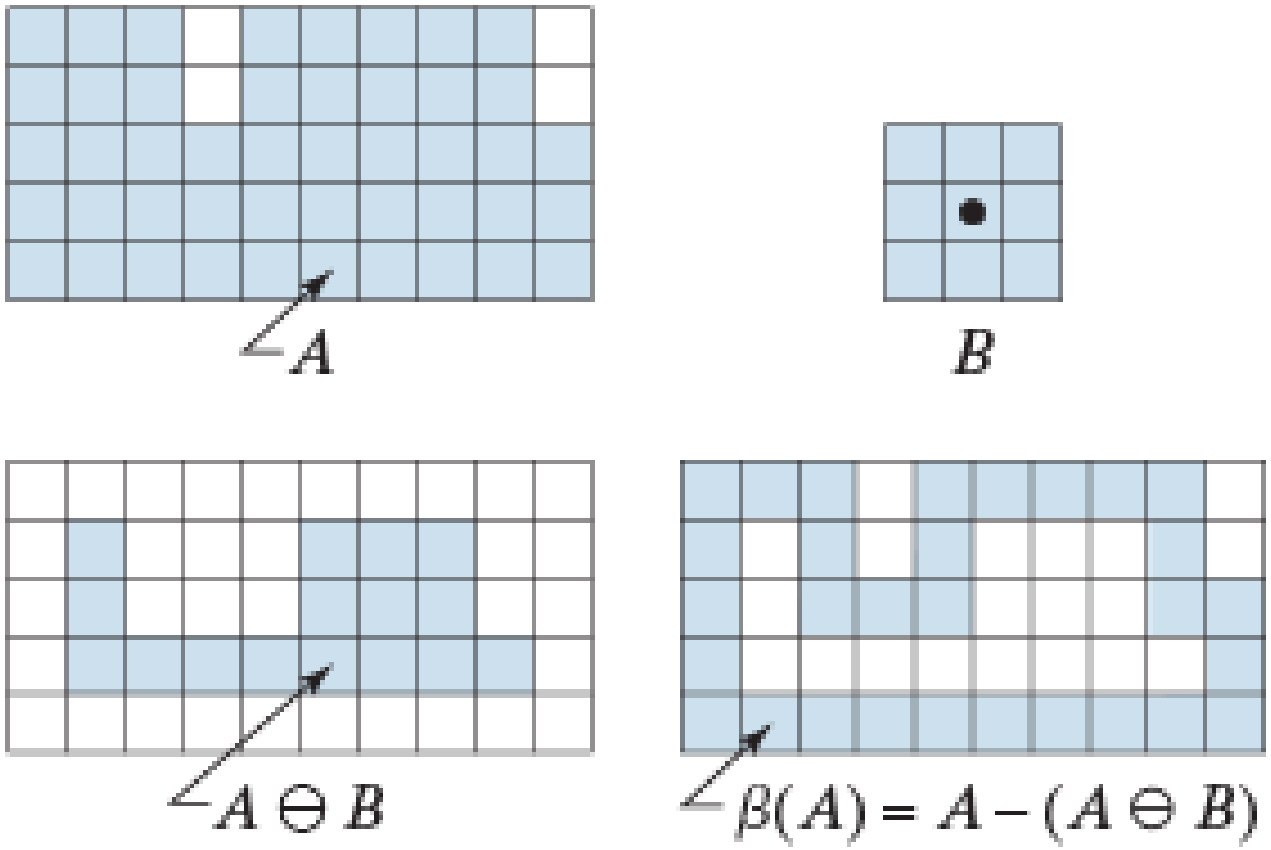
Morphological algorithms

- Boundary extraction
- Hole filling
- Connected components

No assignment or exam problems after here

- Thinning
- Thickening
- Skeletons
- Pruning

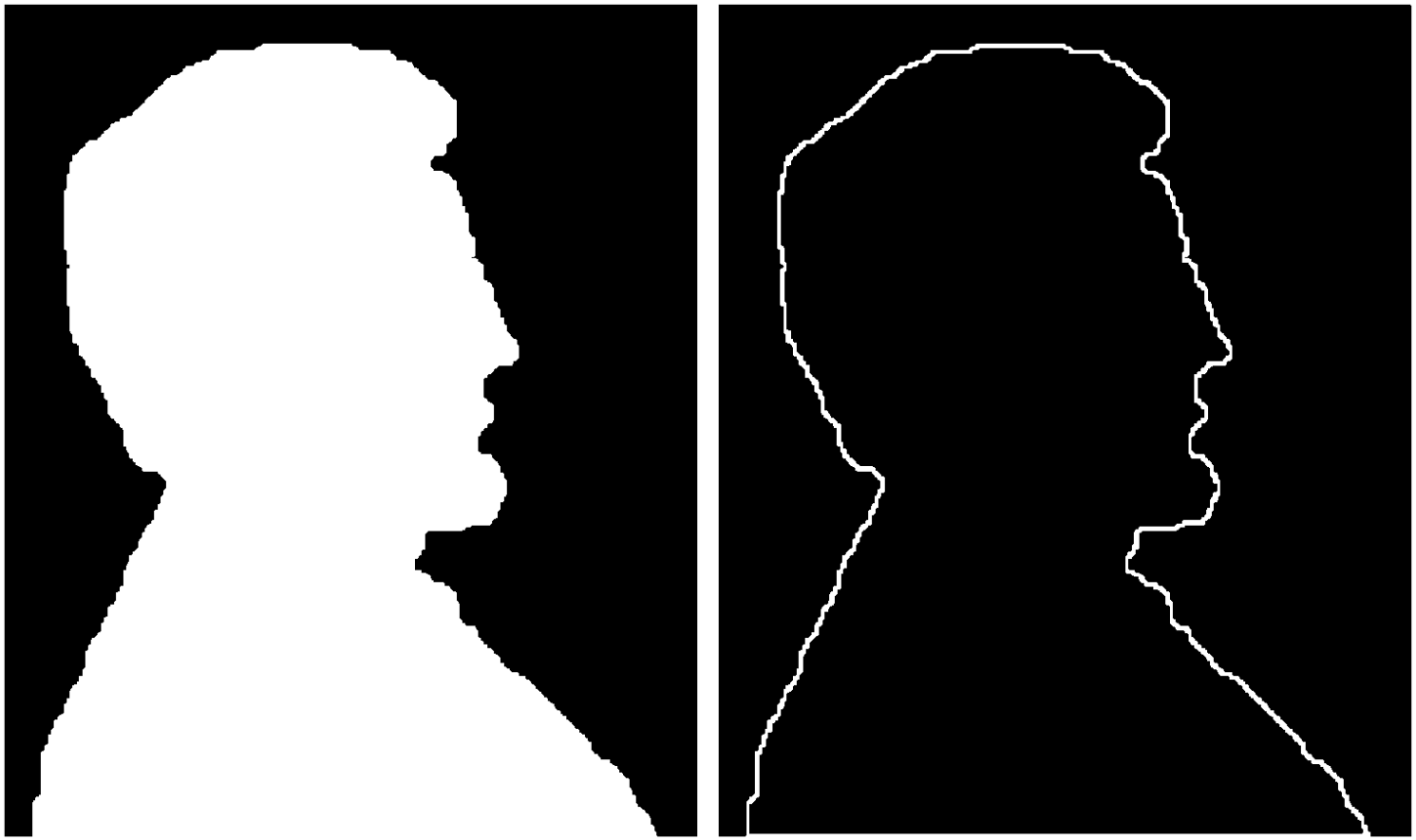
Boundary extraction



Erosion

Set difference

Boundary extraction



Hole filling

X is same size as I

X_0 is all zeros, except ones at a given point in each hole

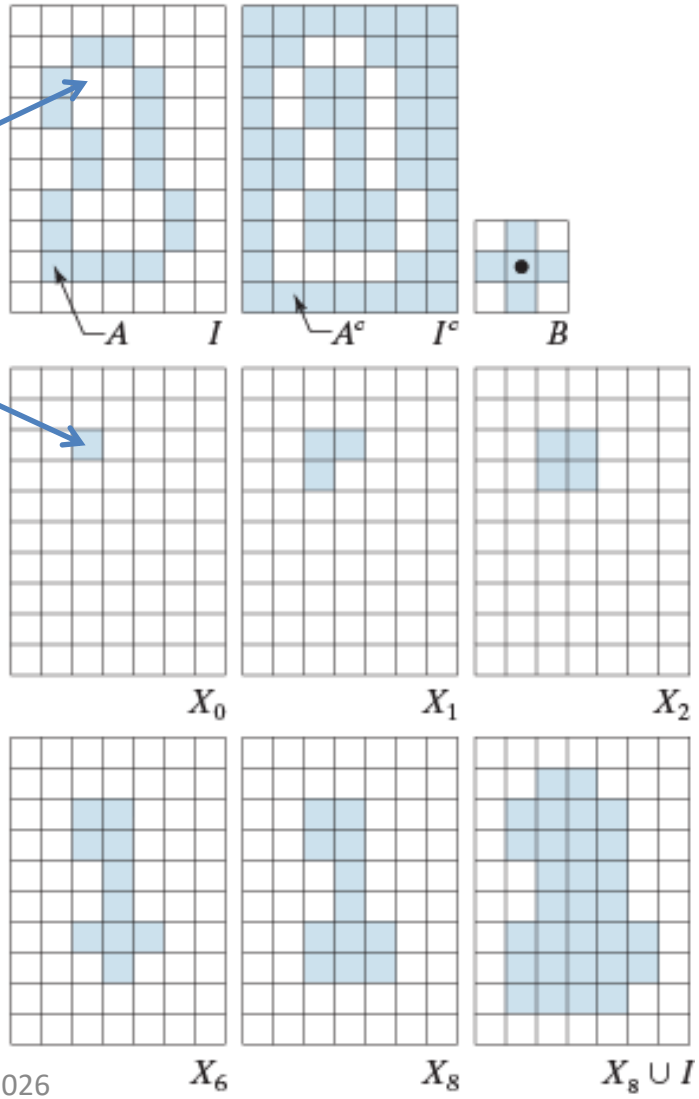
$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, \ldots$

Looks for background points

Intersection with A^c limits dilation

Stop when $X_k = X_{k-1}$

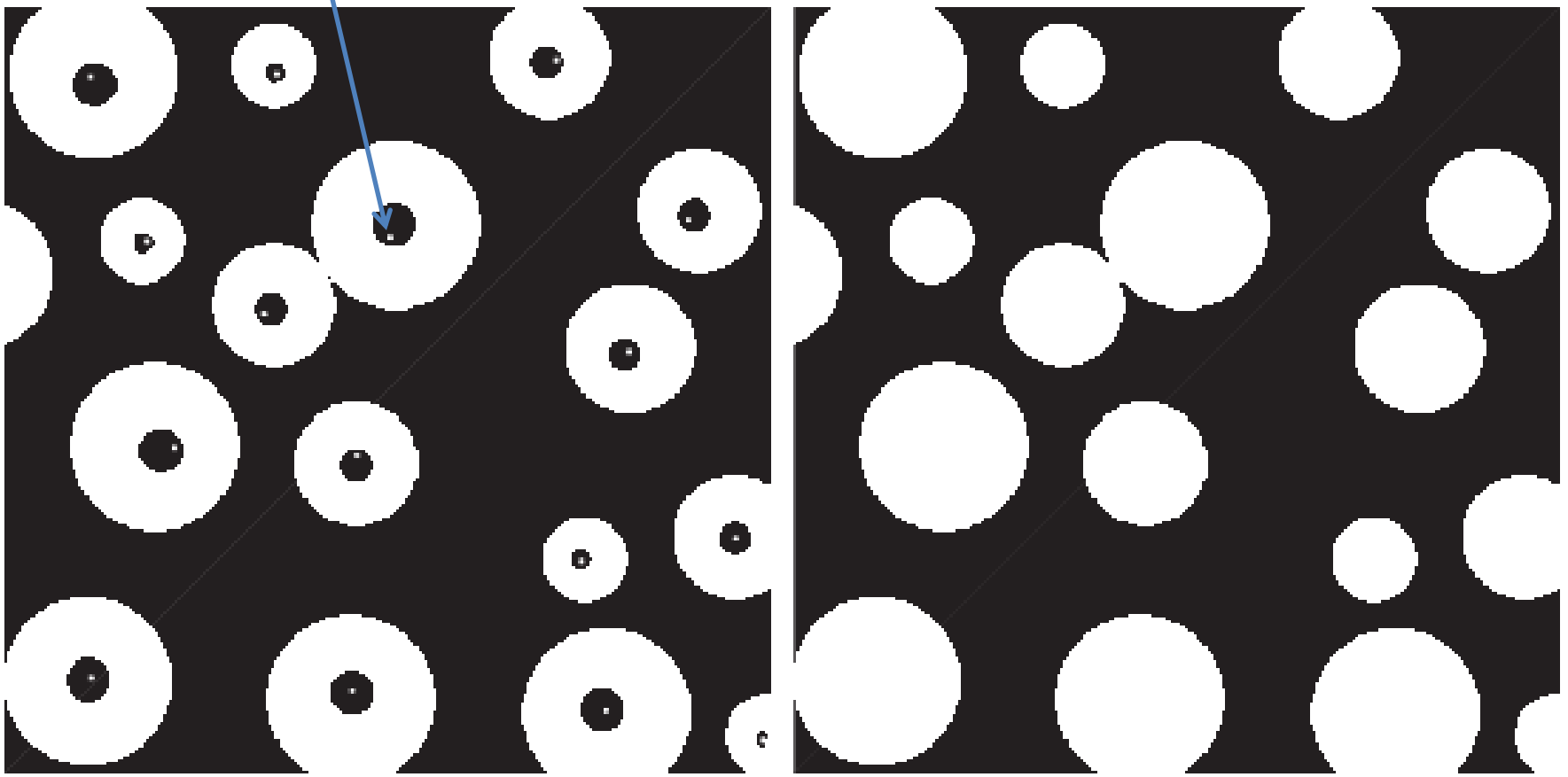
$X_{k_{\text{final}}} \cup A$ is A with holes filled



Hole filling

Given points in holes

All holes filled



Connected components

X is same size as I

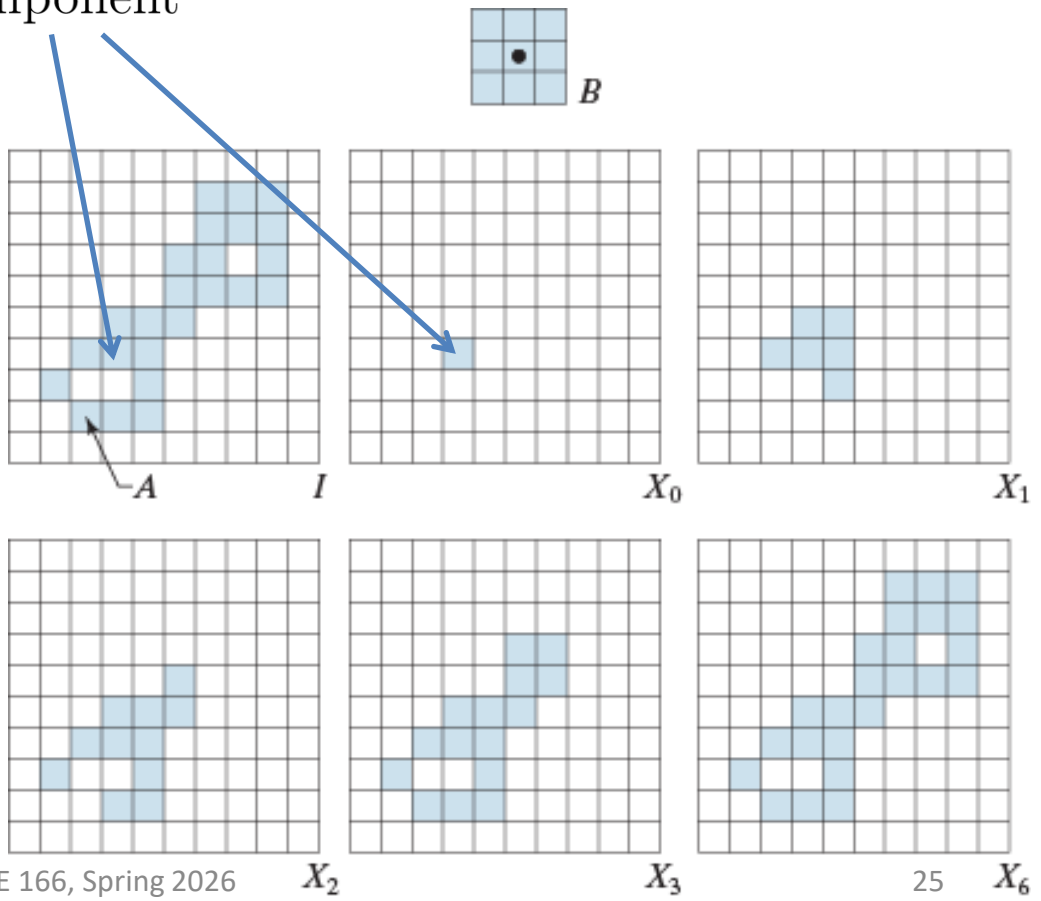
X_0 is all zeros, except ones at a
given point in each connected component

$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, \ldots$

Looks for foreground points

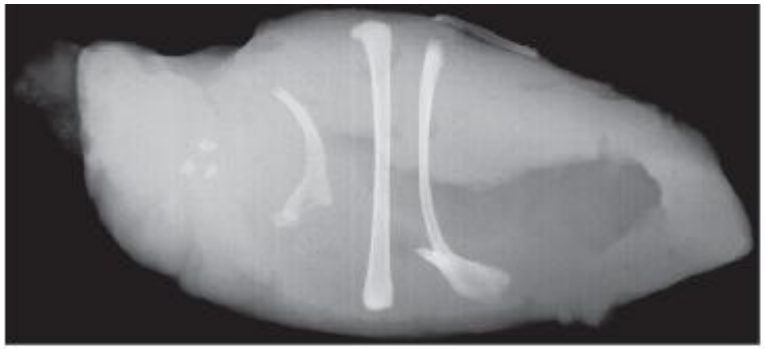
Intersection with A limits dilation

Stop when $X_k = X_{k-1}$



Connected components

X-ray image



Threshold (negative)





Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

15 connected components

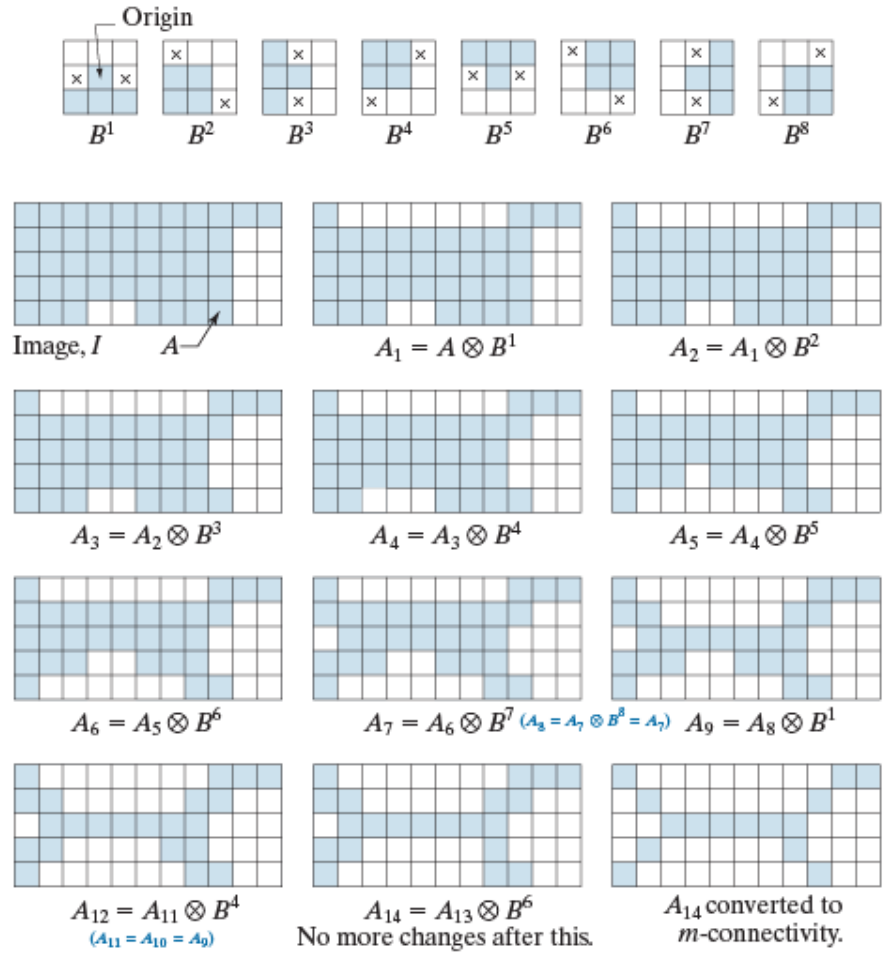
No assignment or exam problems after here

Thinning

A sequence of structuring elements $\{B\} = \{B^1, B^2, \ldots, B^n\}$, including all rotated versions

$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$

Stop when $A_k = A_{k-1}$, where A_k is one pass of B^i



Thickening

Morphological dual of thinning

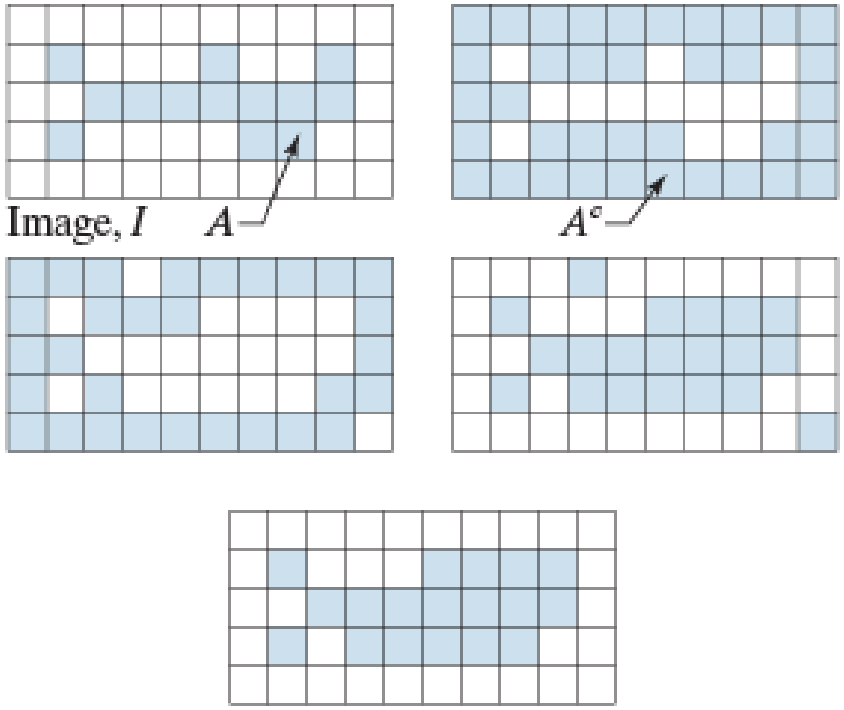
To thicken a set A

$C = A^c$

Thin C

Thick A is C^c

Thick A = Thin A^c



a b
c d
e

FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

Skeletons

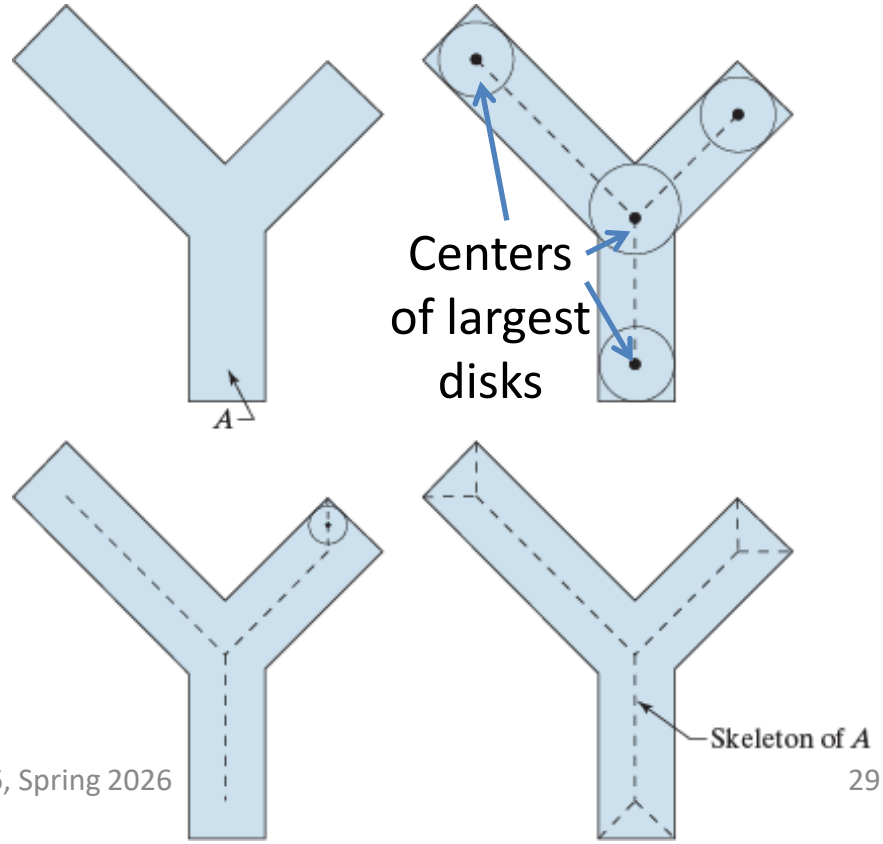
Skeleton of set A

erosion opening

$S(A) = \cup_{k=0}^{K} S_k(A)$, where $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$ are skeleton subsets

$(A \ominus kB)$ is k successive erosions of A

$K = \max\{k | (A \ominus kB) \neq \varnothing\}$ is last iteration before A erodes to an empty set



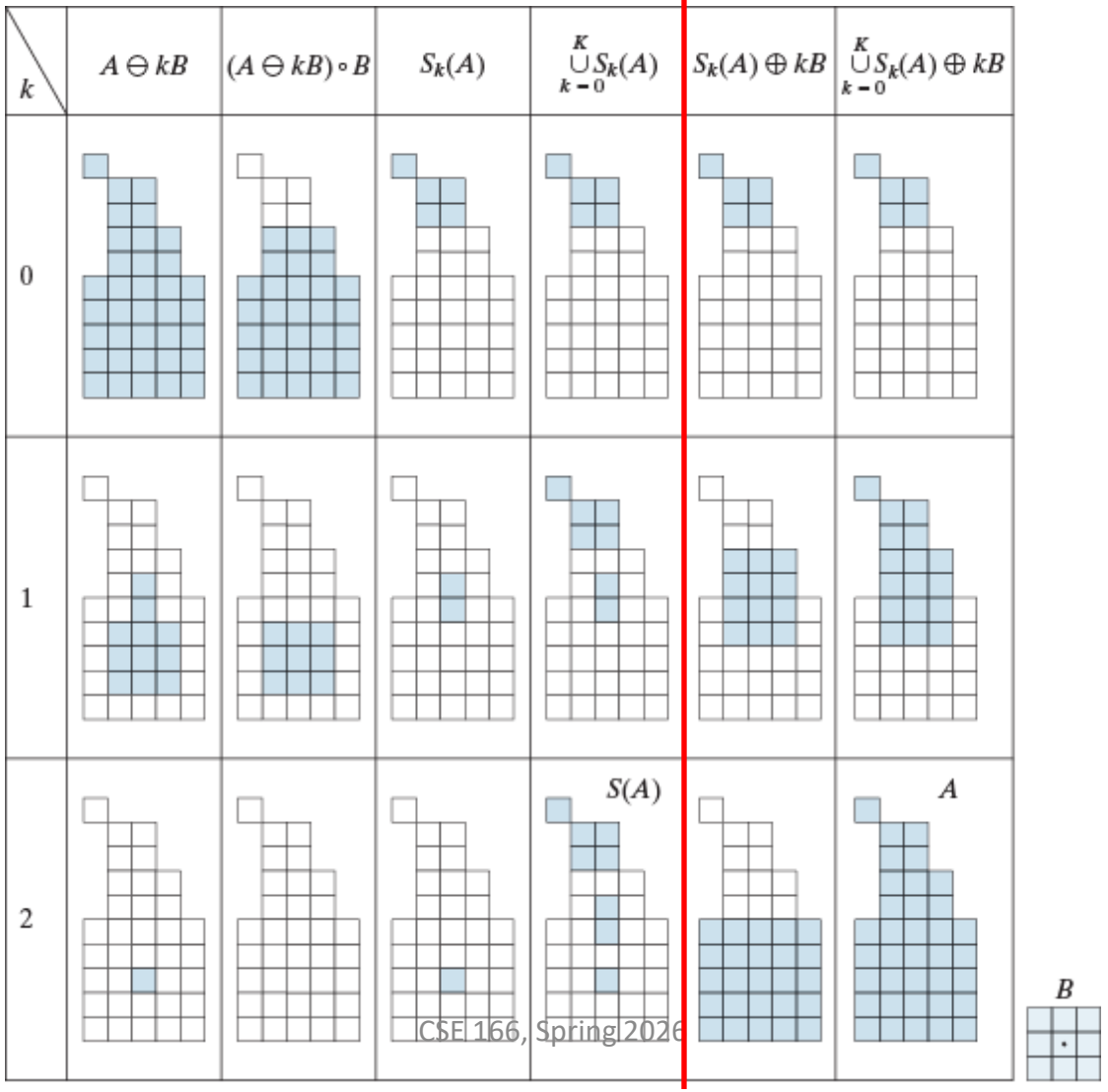
Set A can be reconstructed

$A = \cup_{k=0}^{K} (S_k(A) \oplus kB)$

dilation

Skeletons

Skeleton | Reconstruction



Pruning

Removes "spurs"

$$X_1 = A \otimes \{B\}$$

Hit-or-miss transform:
Exact hit, output true; exact miss, output false

$$X_2 = \cup_{k=1}^{8}(X_1 \circledast B^k)$$

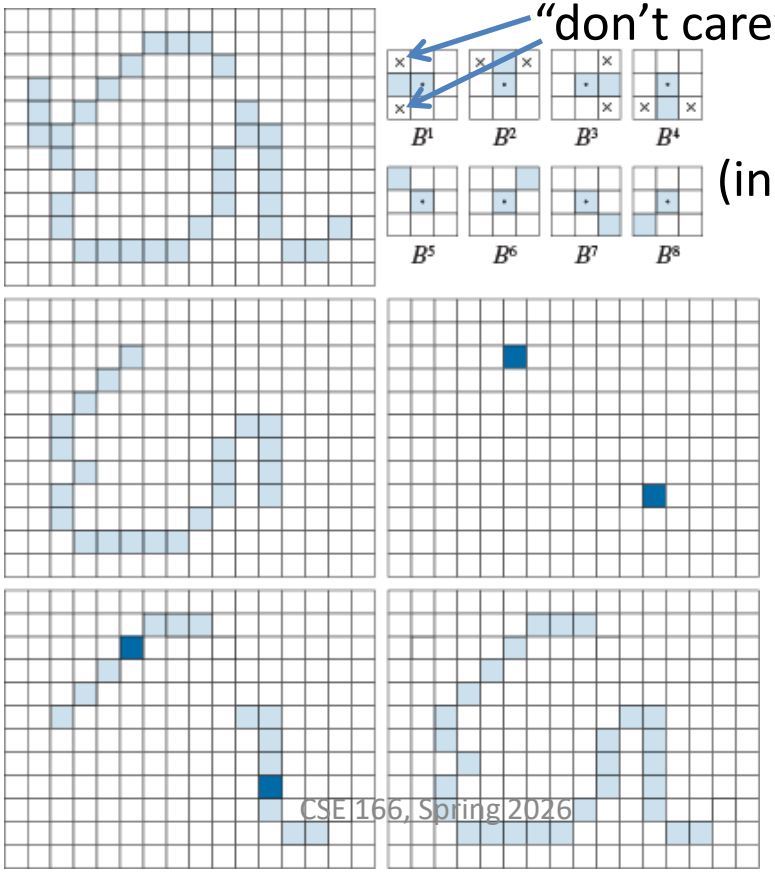
$$X_3 = (X_2 \oplus H) \cap A, \text{ where } H \text{ is } 3 \times 3 \text{ structuring element of ones}$$

$$X_4 = X_1 \cup X_3$$

"don't care" elements

8 structuring elements
(includes all rotated versions)

Character 'a'



a b
c d
e f

FIGURE 9.27
(a) Set A of foreground pixels (shaded).
(b) SEs used for deleting end points.
(c) Result of three cycles of thinning.
(d) End points of (c).
(e) Dilation of end points conditioned on (a).
(f) Pruned image.

Morphological reconstruction

- Geodesic dilation
- Geodesic erosion
- Morphological reconstruction by dilation
- Morphological reconstruction by erosion
- Opening by reconstruction
- Closing by reconstruction
- Hole filling
- Border clearing

Two images

Marker image F

Mask image G

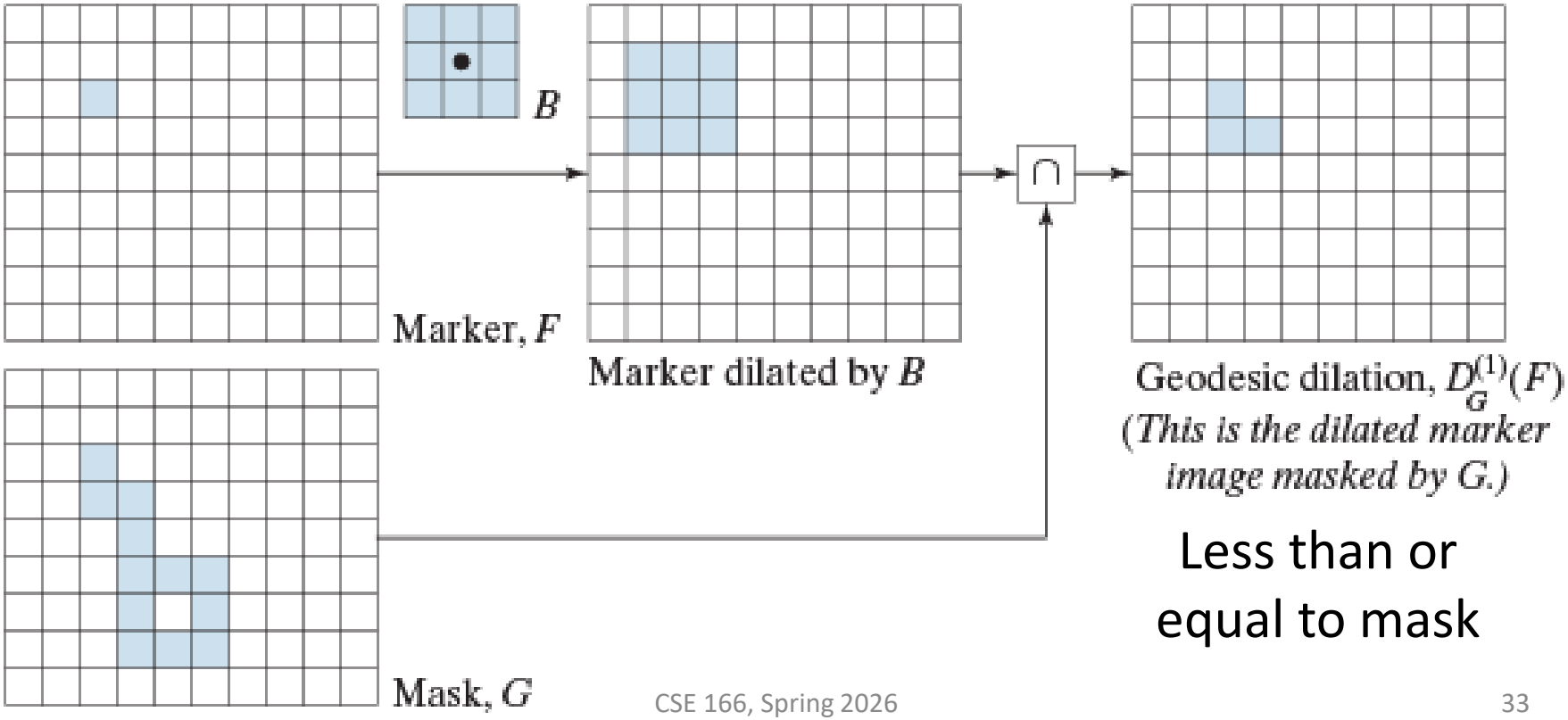
$F \subseteq G$

Geodesic dilation

Intersection limits growth to less than or equal to mask image G

$D_G^{(1)}(F) = (F \oplus B) \cap G$ Geodesic dilation of size 1 of F with respect to G

$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$, where $D_G^{(0)}(F) = F$

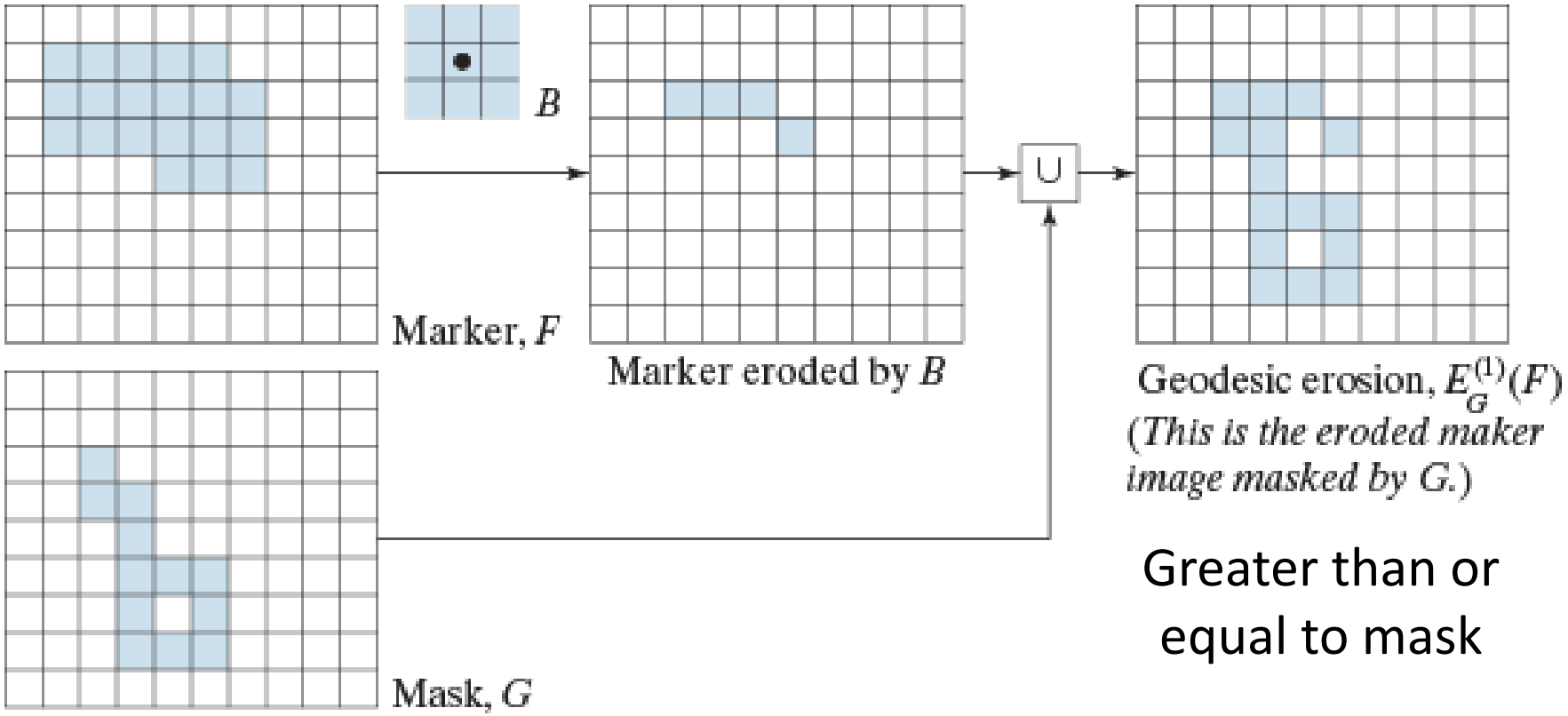


Less than or equal to mask

Geodesic erosion

Union ensures greater than or equal to mask image G

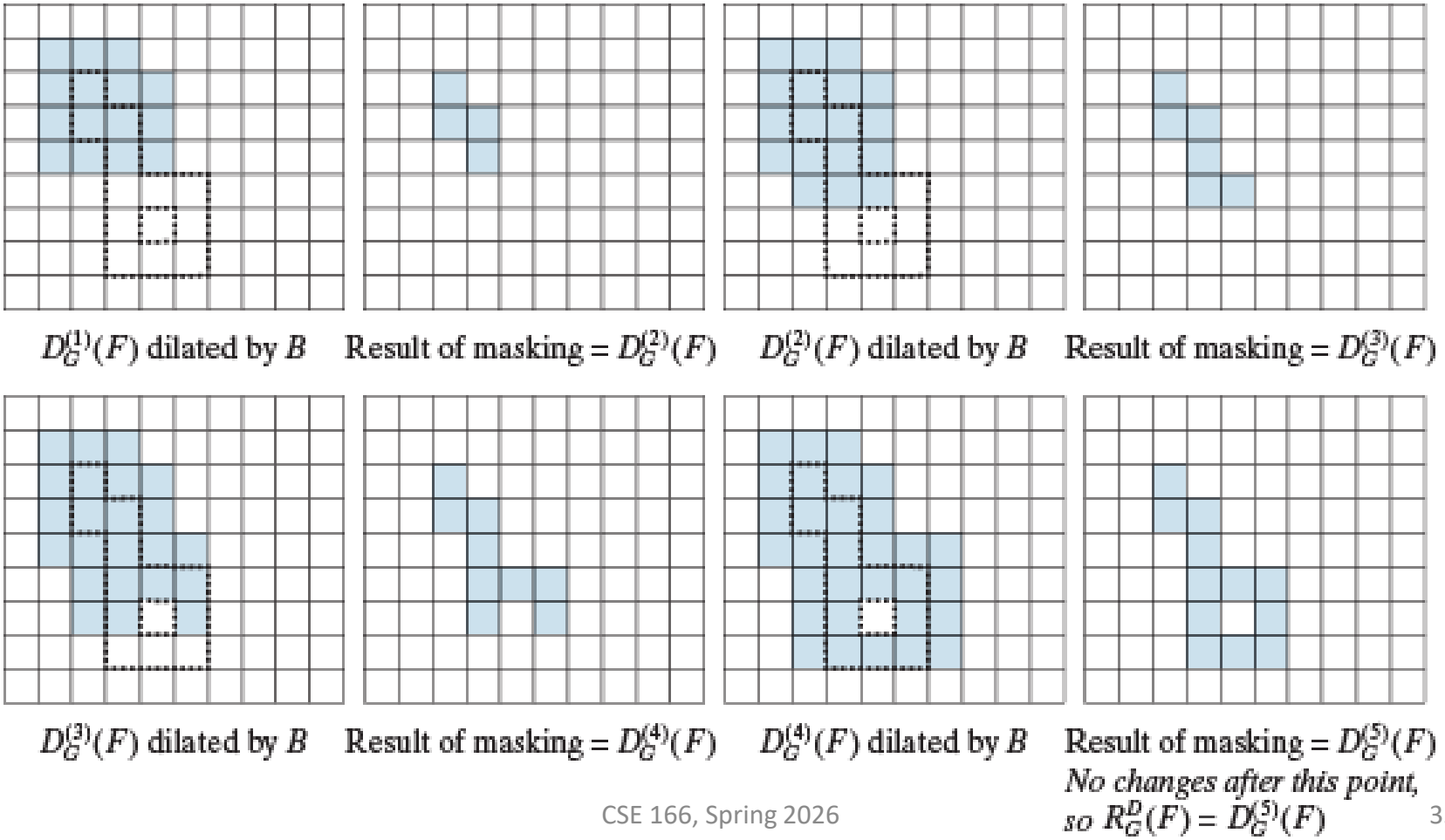
$$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right), \text{ where } E_G^{(1)}(F) = (F \ominus B) \cup G \text{ and } E_G^{(0)}(F) = F$$



Greater than or equal to mask

Morphological reconstruction by dilation

$$R_G^D(F) = D_G^{(k)}(F) \text{ with } k \text{ such that } D_G^{(k)}(F) = D_G^{(k-1)}(F)$$



Opening by reconstruction

$$O_R^{(n)}(F) = R_F^D(\underbrace{F \ominus nB}_{n \text{ successive erosions of } F})$$

Long, vertical strokes

Structuring element: 51x1 pixels

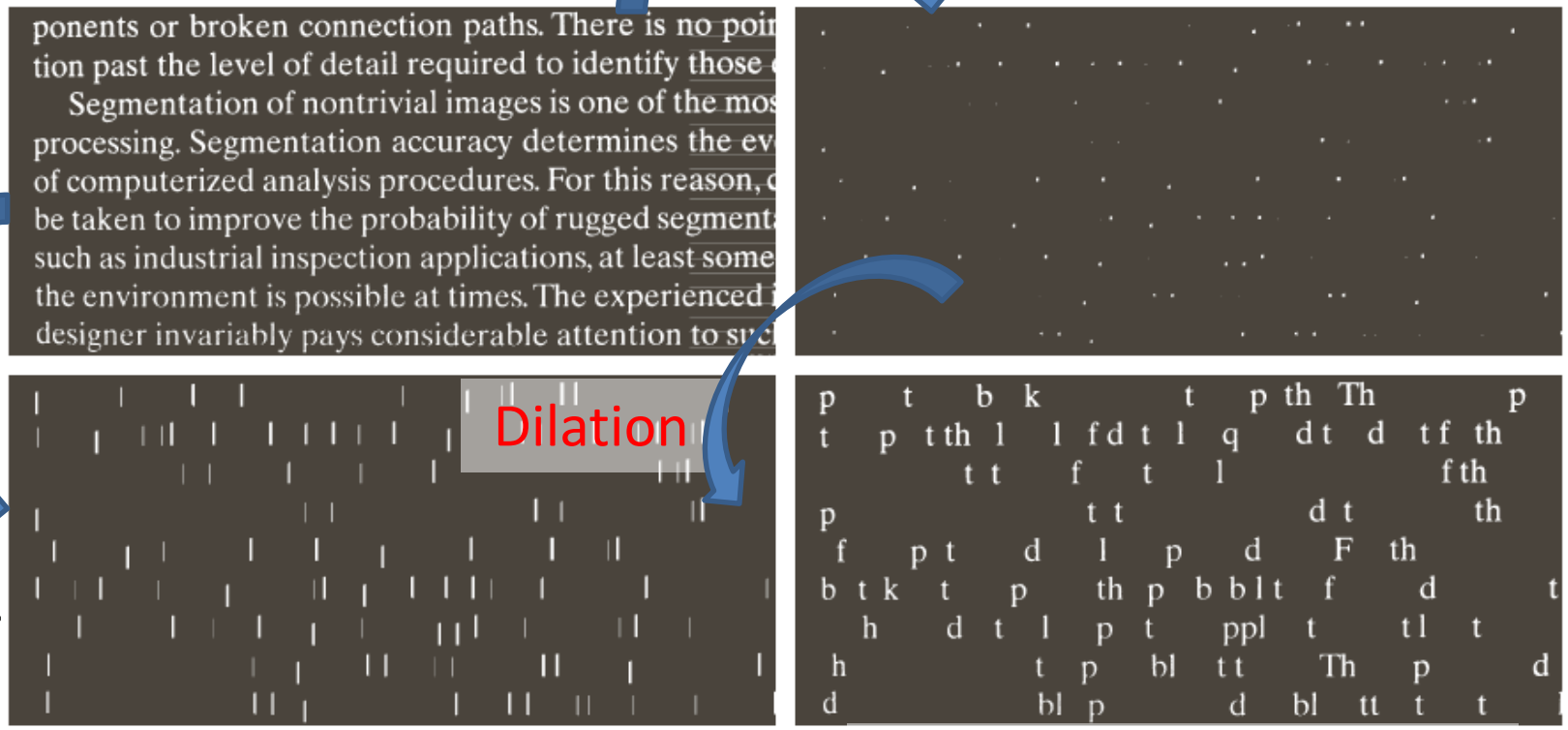
Input and mask image

Erosion

Dilation

Opening

Marker image



Reconstruction results

a b
c d

FIGURE 9.31 (a) Text image of size 918×2018 pixels. The approximate average height of the tall characters is 51 pixels. (b) Erosion of (a) with a structuring element of size 51×1 elements (all 1's). (c) Opening of (a) with the same structuring element, shown for comparison. (d) Result of opening by reconstruction.

Morphological reconstruction

- Morphological reconstruction by erosion

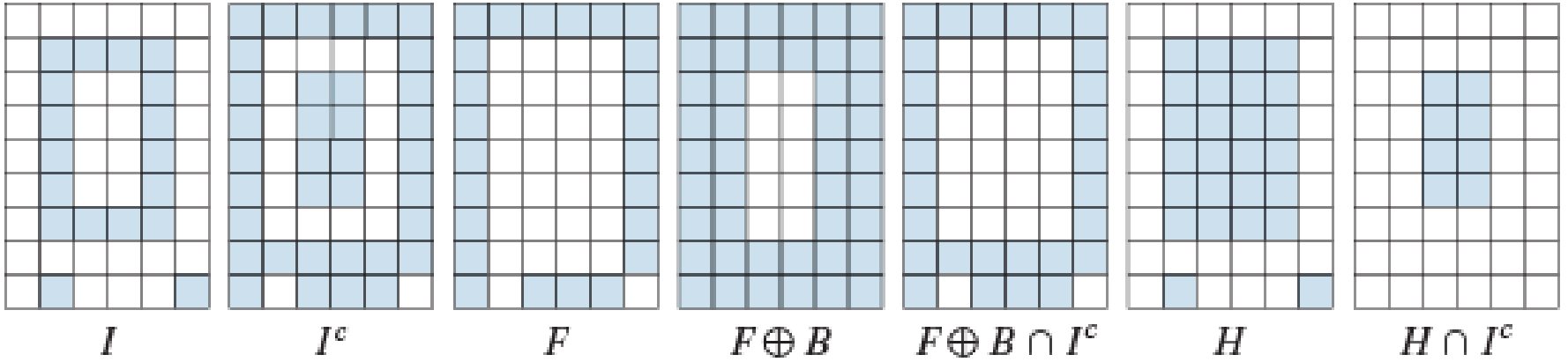
$$R_G^E(F) = E_G^{(k)}(F) \text{ with } k \text{ such that } E_G^{(k)}(F) = E_G^{(k-1)}(F)$$

- Closing by reconstruction

$$C_R^{(n)}(F) = R_F^E(F \oplus nB)$$

Hole filling

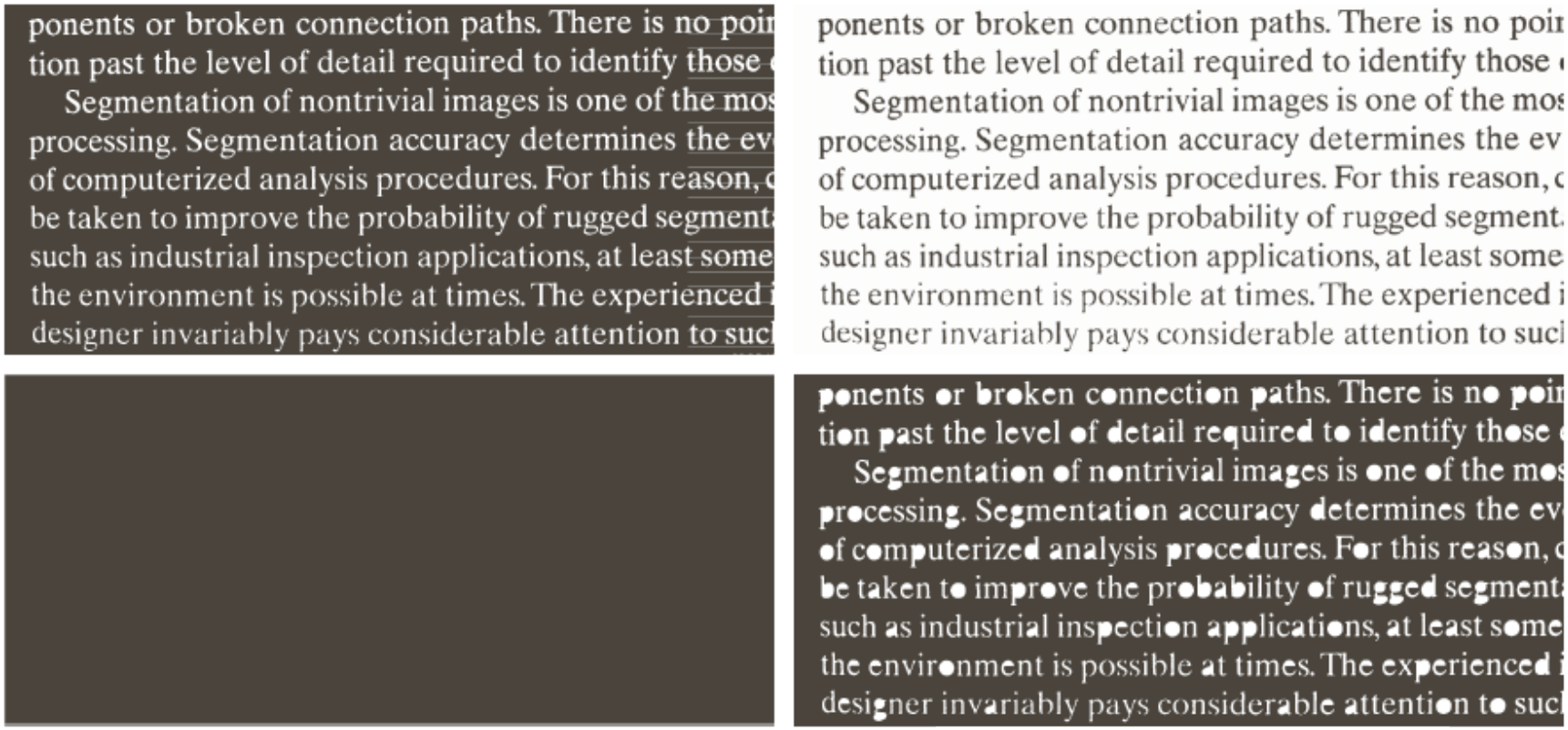
$$H = [R_{I^c}^D(F)]^c$$



Input image with hole filled

Hole

Hole filling



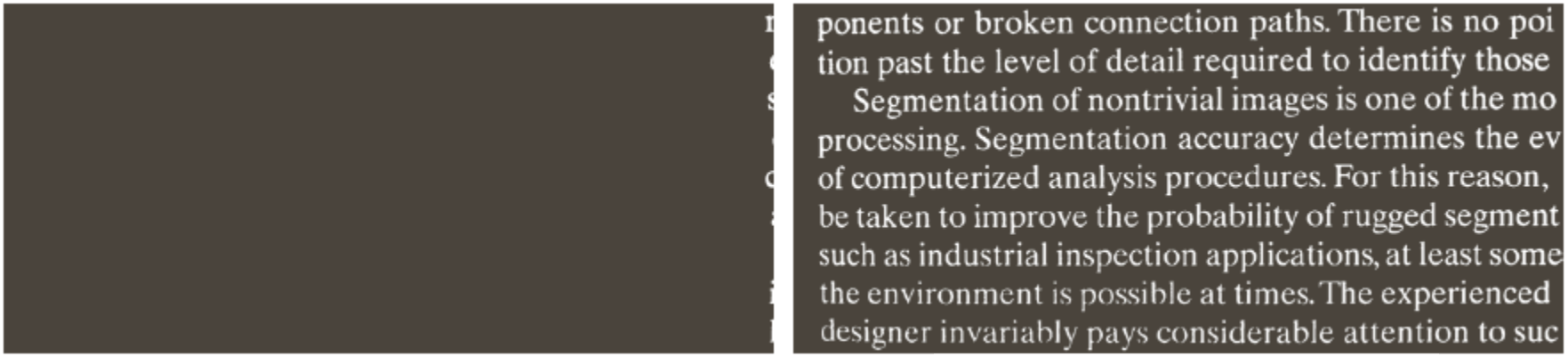
a b
c d

FIGURE 9.33
(a) Text image of size 918 × 2018 pixels.
(b) Complement of (a) for use as a mask image.
(c) Marker image.
(d) Result of hole-filling using Eqs. (9-45) and (9-46).

Input image with holes filled

Border clearing

$$X = I - R_I^D(F)$$



Marker image

Border

Input image with no objects touching the border

a b

FIGURE 9.34
(a) Reconstruction by dilation of marker image. (b) Image with no objects touching the border. The original image is Fig. 9.31(a).

Next Lecture

- Image segmentation
- Reading
  - Chapter 10: Image segmentation I: edge detection, thresholding, and region detection
    - Sections 10.1, 10.2, and 10.3